

COT 3100 Fall 2022 Homework #5
Please Consult WebCourses for the due date/time

1) Determine the following summation in terms of n : $\sum_{i=1}^{2n} (i + 8)$. (Note: Please express your answer in the form $2^a + 2^b + 2^c$, where a , b and c are expressions in terms of n .)

2) Use an index shift to solve the following summation: $\sum_{i=n}^{2n} (2^{i-n})$.

3) Determine the following infinite summation: $\sum_{i=1}^{\infty} (i + 5) \left(\frac{1}{5}\right)^i$.

4) Let $g(n)$ be defined as follows be a function defined on the positive integers as follows:

$$g(1) = 2, g(2) = 2, g(3) = 3$$
$$\text{For all } n > 3, g(n) = 3g(n-1) + g(n-2) + g(n-3).$$

What are the values of $g(4)$, $g(5)$ and $g(6)$?

If you would like for fun, write a computer program which prints out the first 1000 values of $g(n) \bmod 10^9 + 7$. Feel free to include the source code inside the document containing your homework solutions. If you want to have LOTS of fun, write a program that reads in a value of n upto 10^{12} , and quickly produces the value of $g(n)$ when divided by $10^9 + 7$. (Hint: For the latter task, you have to embed the recurrence in a matrix and code up fast matrix exponentiation, which is similar to the fast modular exponentiation taught in class.)

5) Determine the following matrix product, $\begin{bmatrix} n+1 & n \\ -n+1 & n-1 \end{bmatrix} \begin{bmatrix} 1 & n+1 \\ n-1 & 2 \end{bmatrix}$, in terms of n .

6) Let n be a positive integer such that $7 \mid (5^{2n-1} + 2^{2n-1})$. Prove that $7 \mid (5^{2n+1} + 2^{2n+1})$. (Hint: Rewrite $5^{2n+1} + 2^{2n+1}$, where a portion of the expression is 7 times $5^{2n-1} + 2^{2n-1}$, and the rest of the expression is also divisible by 7.)

7) Give a summary of life and work of mathematician Maryam Mirzakhani. Please aim for a length of roughly 200 - 400 words. **Your summary must be typed.** Please state the sources you used in writing your summary.