COT 3100 Section 2 Final Exam - Part A Solution

1) (10 pts) A course grade is comprised of 4 Quizzes, 5 Homework Assignments and 3 Exams. The quizzes are worth 20% of the course grade, the homework assignments are worth 30% of the course grade and the exams are worth the remaining 50% of the course grade. Each of the quizzes is out of 30 points, each of the homework assignments is out of 50 points and each of the exams is out of 100 points. Each quiz is worth the same amount of the course grade, each homework assignment is worth the same amount of the course grade and each exam is worth the same amount of the course grade. Note that a quiz, homework assignment and exam are each worth different amounts of the course grade. Here are Selena's grades on the assignments that have already been turned in:

Quiz 1: 22/30	Homework 1: 45/50	Exam 1: 60/100
Quiz 2: 15/30	Homework 2: 30/50	
Quiz 3: 0/30	Homework 3: 42/50	

As discussed many times in class, Webcourses incorrectly calculates a course grade in the situation when it's known how many assignments in each category there will be for the whole course. We discussed how to properly calculate the course grade in these instances. Thus, as promised, do the following:

(a) Calculate Selena's ACTUAL current course average, which only counts the current assignments, properly weighted.
(b) Selena's grade as it would be calculated on Webcourses.

In order to get any credit on this question, you must show your work and your answers must be perfectly accurate, since incorrect methods will naturally lead to answers that are "pretty close."

Solution

(a) Note that each quiz is worth 5%of the course grade, each homework assignment is worth 6% of the course grade and the exams are worth (50/3)% of the course grade. It follows that each quiz point is (1/6)% of the course grade, each homework point is (6/50)% of the course grade and each exam point is (1/6)% of the course grade. The actual course grade, with 15% + 18% + 50/3% = (149/3)% of the total course grade is calculated as follows:

$$\frac{(22+15+0)\left(\frac{1}{6}\right)+(45+30+42)\left(\frac{6}{50}\right)+60\left(\frac{1}{6}\right)}{\frac{149}{300}}=\frac{300\left(\frac{37}{6}+\frac{351}{25}+10\right)}{149}=\mathbf{\frac{9062}{149}}$$

Written as a regular percentage, rounded to two decimal spots, we get **60.82%**

(b) Webcourses calculates the percentage in each category and weights those categories based on the final weighting percentages. So the quiz percentage is $\frac{37}{90}\times 100=\frac{370}{9}$, the total homework percentage is $\frac{117}{150}\times 100=78\%$ and the total exam percentage is 60%. Thus, Webcourses would do the following calculation:

$$.2\times\frac{370}{9}+.3\times 78+.5\times 60=\frac{74}{9}+\frac{234}{10}+30=\frac{740+2106+2700}{90}=\mathbf{\frac{2773}{45}}$$

Written as a regular percentage, rounded to two decimal spots, we get **61.62%**

Grading: 5 pts for both averages, give partial credit as you see fit. There are lots of equivalent algebraic ways to do what I show above, so be careful to give credit to alternative ways that work.

2) (10 pts) Let r and s be the roots of the quadratic equation $x^2 - 13x + 9 = 0$. Find the quadratic equation with the roots 2r+s and r + 2s with a leading coefficient of 1. (Note: Full credit will only be given for the technique where the value of neither r nor s is ascertained.)

Solution

Given that the roots of the first equation are r and s, using the formula for the sum and product of the roots, we have that

$r + s = 13$
$rs = 9$

Now, we want to find the sum and product of the roots of a quadratic whose roots are 2r+s and r+2s:

$(2r + s) + (r + 2s) = 3r + 3s = 3(r+s) = 3(13) = 39$

$(2r + s)(r + 2s) = 2r^2 + 5rs + 2s^2 = 2r^2 + 4rs + 2s^2 + rs = 2(r^2 + 2rs + s^2) + rs = 2(r + s)^2 + rs = 2(13)^2 + 9 = 347$.

It follows that the desired quadratic equation is **$x^2 - 39x + 347 = 0$.**

Grading: 2 pts for writing down what r+s is and what rs is.
1 pt for stating that we need to know (2r+s) + (r+2s)
1 pt for stating that we need to know (2r+s) * (r+2s)
1 pt for finding (2r+s) + (r+2s)
4 pts for finding (2r+s) * (r+2s)
1 pt for giving the final quadratic

3) (10 pts) For this question, assume that if x_1 ounces of liquid 1 at temperature t_1 is mixed with x_2 ounces of liquid 2 at temperature t_2, the resulting mixture is $x_1 + x_2$ ounces with a temperature of $\frac{t_1x_1+t_2x_2}{x_1+x_2}$. Arup's wife, Anita, likes her tea mixed with milk. (She also likes it with sugar, but I didn't want to make this problem too hard!) Unfortunately, she has a very specific preference for both the ratio of tea to milk and the temperature of the overall mixture. Specifically, she wants a 6:1 ratio of tea to milk at a temperature of 180° F. On Arup's first attempt, he mixed 8 ounces of tea at 200° F with half an ounce of milk at 40° F. It's not possible for Arup to add any more tea to the mixture, but he is allowed to add more milk and he can heat that milk to any temperature greater than 40° F, if necessary, before he adds it. How much milk should Arup add to the tea and at what temperature should he heat it before adding it, to fix the tea?

Solution

Since the mixture must have 8 ounces of tea and the ratio of tea to milk is 6:1, it follows that the total amount of mile to add to the mixture is 8/6 = 4/3 ounces. Currently 1/2 an ounce was added, so the total amount of milk that must be added is 4/3 - 1/2 = 5/6 ounces.

The current temperature of the mixture is $\frac{200(8)+40(.5)}{8.5} = \frac{1620}{8.5} = \frac{3240}{17}$ degrees Fahrenheit.

Let t be the temperature of the 5/6 added ounces of milk. We know we want this temperature to also equal 180 degrees Fahrenheit. Set these two expressions equal to one another:

$$\frac{\frac{3240}{17} \times \frac{17}{2} + t \times \frac{5}{6}}{\frac{28}{3}} = 180$$

$$\frac{3(1620 + t \times \frac{5}{6})}{28} = 180$$

$$4860 + \frac{5t}{2} = 5040$$

$$\frac{5t}{2} = 180$$

$$t = 180 \times \frac{2}{5} = 72$$

It follows that Arup should add **<u>5/6th</u>** of an ounce of milk to the tea, heating it to a temperature of **<u>72° F,</u>** before adding it.

Grading: 3 pts for figuring out that 5/6 ounces of milk needs to be added.
3 pts for figuring out that the current temperature is $\frac{3240}{17}$.
4 pts for figuring out the final answer.

Note: You can skip part 2 by solving this $\frac{200\times 8+40\times\frac{1}{2}+t\times\frac{5}{6}}{\frac{19}{2}} = 180$. If they do this, then 7 pts for solving this equation.

COT 3100 Section 2 Final Exam - Part B (Logic, Sets, Relations, Functions) - 30 pts Solutions

1) (8 pts) **Use a truth table** to show that the Rule of Resolution on the Rules of Inference list is a one way implication. Namely, prove that the logical expressions to the left and right of the implication sign are **NOT** equivalent. Your table should have seven columns.

Solution

p	q	r	$p \vee q$	$\bar{p} \vee r$	$(p \vee q) \wedge (\bar{p} \vee r)$	$q \vee r$
F	F	F	F	T	F	F
F	F	T	F	T	F	T
F	T	F	T	T	T	T
F	T	T	T	T	T	T
T	F	F	T	F	F	F
T	F	T	T	T	T	T
T	T	F	T	F	F	T
T	T	T	T	T	T	T

Grading: 1 point per row, the row has to be all correct to get the point. If a column is missing, take off 1 pt total for the column being missing.

2) (7 pts) Let A, B and C be arbitrary sets. **Without the aid of a membership table,** prove:

$$if\ A - B = C - B, then\ A \cup B = C \cup B$$

Note: The proof is long since you have to show that both the LHS is a subset of the RHS and that the RHS is a subset of the LHS. But, the second part of the proof is symmetric to the first (since swapping the roles of A and C doesn't change the problem at all). Thus, just prove the first part and note that the symmetry proves the other subset relation.

Solution

We prove the equality of sets by showing that the subset relationship holds in both directions, namely, we must prove that:

(a) $if\ A - B = C - B, then\ A \cup B \subseteq C \cup B$, and
(b) $if\ A - B = C - B, then\ C \cup B \subseteq A \cup B$

Notice that both statements are symmetric, since the roles of A and C can be switched in the first to produce the second statement. Thus, if we can prove part (a), that serves as a proof of part (b) and we can conclude that the sets $A \cup B$ and $C \cup B$ are equal, as desired.

To prove that one set is a subset of another, we must show that an arbitrarily chosen element in the first set belongs to the second set. Let x be an arbitrarily chosen element of $A \cup B$. Namely, let $x \in A \cup B$. We aim to prove that $x \in C \cup B$.

It's given that $x \in A \cup B$.
By definition of set union, either we have that $x \in A$ or $x \in B$. (Of course, it's possible that both are true, but we know that at least one must be true.)

If the latter is true, by the definition of union, it follows that $x \in C \cup B$.

Thus, we must only consider the case where $x \notin B$, since we've already proven the claim for all $x \in B$. In this case, since we know that $x \in A$ or $x \in B$ is true, it follows that $x \in A$ (Disjunctive Syllogism). Thus, we have that $x \in A$ and $x \notin B$. By definition of set different, we know that $x \in A - B$. Since it's given that $A - B = C - B$, it follows that $x \in C - B$, by definition of set equality. Then, by definition of set difference, we have that $x \in C$ and $x \notin B$. Since $x \in C$, it this case, by definition of union, it follows that $x \in C \cup B$, as desired.

Thus, we've proven in all cases of the arbitrarily chosen element x, that $x \in C \cup B$, proving the assertion for all x.

Grading: Starting with arbitrary element - 1 pt
Stating there are two cases, x in A or x in B - 1 pt
Handling proof for x in B - 1 pt
Showing that only case left is x in A and not in B - 1 pt
Showing in this case that x is in C - B, so that it's in C - 2 pts
Concluding the proof that x is in C union B - 1 pt

3) (10 pts) Let R be the following relation defined over the set of integers:

$$R = \{ (a, b) \mid (a + b) \equiv 0 \pmod{10} \}$$

Is R reflexive, irreflexive, symmetric, anti-symmetric or transitive?

Please give your answer (yes/no) for each of the five parts and give a brief justification for each. Each answer is worth 1 pt and each justification is worth 1 pt. You can not earn the point for the justification if the answer is incorrect.

Solution

The relation is NOT reflexive, since $(3,3) \notin R$, because $3 + 3 \equiv 6 \pmod{10}$.

The relation is NOT irreflexive since $(5,5) \in R$, because $5 + 5 \equiv 0 \pmod{10}$.

The relation IS symmetric. Let (a, b) be an arbitrary element of the relation. Then we know that $(a + b) \equiv 0 \pmod{10}$, but it also follows from this that $(b + a) \equiv 0 \pmod{10}$, since $a + b = b + a$. The latter statement infers that (b, a) is in the relation, as desired.

The relation is NOT anti-symmetric because both (3, 7) and (7, 3) are part of it.

The relation is NOT transitive because although both (3, 7) and (7, 3) are elements of it (3, 3) is not an element of it.

Grading: 1 pt for each answer. If answer is correct, then 1 pt for the reasoning.

4) (5 pts) Let $f(x) = x^3 + ax^2 + bx + c$. It is known that $f(-1) = 84$, $f(0) = 72$ and $f(1) = 50$, what are the values of a, b and c?

Solution

$f(0) = 0^3 + a(0)^2 + b(0) + c = c = 72$, so **$c = 72$.**

$f(1) = 1^3 + a(1)^2 + b(1) + 72 = a + b + 73 = 50$

$f(-1) = (-1)^3 + a(-1)^2 + b(-1) + 72 = a - b + 71 = 84$

Adding these two equations we get:

$$2a + 144 = 134$$
$$2a = -10$$
$$\mathbf{a = -5}$$

Plugging into the first equation and solving for b we find:

$$-5 + b + 73 = 50$$
$$b + 68 = 50$$
$$\mathbf{b = -18}$$

it follows that $f(x) = x^3 - 5x^2 -18x + 72$. Incidentally, this factors nicely: $f(x) = (x-3)(x+4)(x-6)$.

Grading: 1 pt for figuring out c
2 pts for plugging in x = 1, x = -1
1 pt for value of a
1 pt for value of b

COT 3100 Section 2 Final Exam - Part C (Number Theory, Induction) - 30 pts Solutions

1) (5 pts) Let n = 2^9k, where k is a positive odd integer. What percentage of n's divisors are even?

Solution

Let k have d divisors. Then n has 10d divisors, since any divisor of k can be multiplied by 2^0 or 2^1 or 2^2, or …, 2^9 to create all the divisors of n. Of this list of divisors, only the ones which are of the form 2^0d', where d' is a divisor of k, are odd. This is precisely d of those divisors. It follows that 9d divisors of n are even, and 9d out of a total of 10d is **90% of the divisors.**

Grading: 2 pts for stating in some way that there are 10 possible exponents to 2 in the divisors of n, 2 pts for observing that each of these is equally likely in a randomly chosen divisor of n, 1 pt for concluding that 90% is the answer from these observations.

2) (5 pts) Let gcd(a, b) denote the greatest common divisor of positive integers a and b and let lcm(a, b) denote the least common multiple of positive integers a and b. Let a, b and c be arbitrary positive integers. Simplify the following expression so that your final answer is solely in terms of a, b and c, and does not contain any reference to the gcd or lcm of pairs of integers:

$$\frac{\gcd(a,b)\gcd(b,c)lcm(a,b)lcm(b,c)}{\gcd(a,c)lcm(a,c)}$$

Solution

Recall in class that we proved for any positive integers a and b, that gcd(a, b)lcm(a, b) = ab, use this formula and substitute:

$$\frac{\gcd(a,b)\gcd(b,c)\, lcm(a,b)lcm(b,c)}{\gcd(a,c)\, lcm(a,c)} = \frac{\gcd(a,b)\,\mathrm{lcm}(a,b)\, gcd(b,c)lcm(b,c)}{\gcd(a,c)\, lcm(a,c)} = \frac{(ab)(bc)}{(ac)} = \boldsymbol{b^2}$$

Grading: 2 pts for stating the fact about gcd, lcm, 3 pts for substituting that fact and simplifying.

3) (10 pts) Let F_n denote the n^{th} Fibonacci number. (Recall $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for all integers $n > 2$.) Using induction on n, prove the following summation formula for all positive integers n:

$$\sum_{i=1}^{2n-1} F_i F_{i+1} = F_{2n}^2$$

Solution

Base case: n = 1, LHS = $\sum_{i=1}^{2(1)-1} F_i F_{i+1} = F_1 F_2 = 1 \times 1 = 1$

RHS = $F_{2(1)}^2 = F_2^2 = 1^2 = 1$

Inductive hypothesis: Assume for an arbitrarily chosen positive integer n = k that

$$\sum_{i=1}^{2k-1} F_i F_{i+1} = F_{2k}^2$$

Inductive step: Prove for n = k + 1 that

$$\sum_{i=1}^{2(k+1)-1} F_i F_{i+1} = F_{2(k+1)}^2$$

$\sum_{i=1}^{2(k+1)-1} F_i F_{i+1} = \sum_{i=1}^{2k+1} F_i F_{i+1}$

$= \left(\sum_{i=1}^{2k-1} F_i F_{i+1}\right) + F_{2k} F_{2k+1} + F_{2k+1} F_{2k+2}$, splitting the sum

$= F_{2k}^2 + F_{2k} F_{2k+1} + F_{2k+1} F_{2k+2}$, using the IH

$= F_{2k}(F_{2k} + F_{2k+1}) + F_{2k+1} F_{2k+2}$, factoring F_{2k} from first 2 terms

$= F_{2k}(F_{2k+2}) + F_{2k+1} F_{2k+2}$, using the Fibonacci recurrence

$= F_{2k+2}(F_{2k} + F_{2k+1})$, factoring F_{2k+2} from the 2 terms

$= F_{2k+2}(F_{2k+2})$, using the Fibonacci recurrence

$= F_{2k+2}^2$, definition of square…

Grading: 1 pt base case, 1 pt IH, 2 pts IS, 1 pt split sum, 2 pts use IH, 3 pts rest of the algebra, give partial as you see fit.

4) (10 pts) Let *a* be an odd integer. Prove for all positive integers *n* via induction that $a^{2^n} - 1$ is divisible by 2^{n+2}.

Solution

Base case: n = 1, we must show that $a^{2^1} - 1 = a^2 - 1 = (a-1)(a+1)$ is divisible by 2^{1+2} = 8.

Since a is odd, there exists an integer z such that a = 2z + 1. Substitute to get

$$(a-1)(a+1) = (2z+1-1)(2z+1+1) = (2z)(2z+2) = 4z(z+1)$$

Now, note that z must be either even of odd. If z is even z(z+1) is even. If z is odd, then z+1 is even and it also follows that z(z+1) is even. Thus, we can conclude that z(z+1) is even. Thus, for some integer y, z(z+1) = 2y, substituting we get:

$$4z(z+1) = 4(2y) = 8y = 2^3 y$$

This proves that the desired quantity is divisible by 8, proving that the given statement is true for the base case, n = 1.

Inductive hypothesis: Assume for an arbitrarily chosen positive integer n = k that $a^{2^k} - 1$ is divisible by 2^{k+2}.

Inductive step: prove for n = k + 1 that $a^{2^{k+1}} - 1$ is divisible by 2^{k+3}.

$a^{2^{k+1}} - 1 = (a^{2^k} + 1)(a^{2^k} - 1)$, using the difference of squares factoring formula

Now, utilizing the inductive hypothesis, $2^{k+2} | (a^{2^k} - 1)$. It follows that there exists an integer c such that $a^{2^k} - 1 = c(2^{k+2})$. Substitute accordingly:

$$= (a^{2^k} + 1)c(2^{k+2})$$

Since a is odd, any positive integer power of a is odd as well, thus a^{2^k} is odd and $a^{2^k} + 1$ is even. Thus, by definition of even numbers, there exists some integer d such that $a^{2^k} + 1 = 2d$. Substitute:

$$= (2d)c(2^{k+2})$$
$$= (2^{k+3})(cd)$$

Since c and d are integers and we've expressed the quantity in question as a multiple of 2^{k+3}, it follows that $a^{2^{k+1}} - 1$ is divisible by 2^{k+3}, as desired, proving the inductive step.

We can conclude that the given statement is true for all positive integers n.

Grading: 5 pts base case, 1 pt IH, 1 pt IS, 1 pt factor, 1 pt use IH, 1 pt stating last term is odd and concluding

COT 3100 Section 2 Final Exam - Part D (Counting, Probability) - 35 pts Solutions

1) (5 pts) A subsequence of a sequence is any sequence that can be obtained from the original sequence by deleting 0 or more of the items and keeping all the items in the same order. For example, the sequence A, B, C, D, E, F has the subsequence B, C, F, obtained by deleting A, D and E. How many non-empty subsequences are there of the sequence Q, U, A, K, E, R?

Solution

Each subset of the letters corresponds to a subsequence and since each of the letters are distinct, each subsequence is distinct. It follows that there are **$2^6 - 1 = 63$** non-empty subsequences (the total number of subsets of 6 items minus the empty set.)

Grading: Mostly all or nothing. 5 pts for 63 as long as the reason is given 4 pts for 64 (take off 1 for forgetting the -1). It's hard to see other answers that are worth any credit, bit if you see something, feel free to award a bit of partial credit.

2) (10 pts) Jabba the Hut lives in New York City and loves to eat. We can model New York City as the Cartesian grid. Naturally, each street corner (intersection of a North-South Avenue and East-West Street) is a lattice point on the grid. Jabba lives at (0, 0) and goes to work every morning, which is located at (12, 10), which is 12 blocks east and 10 blocks north of his home. He walks 22 blocks each day to work. (Walking a block means starting at the lattice point (x, y) and either walking east one block to (x+1, y) or north one block to (x, y+1).) Each street corner, except Jabba's home, (0, 0), has a hot dog stand and a pretzel stand. If Jabba arrives at a corner by walking north, he always buys a pretzel at the corner when he arrives, and if he arrives at the corner by walking east, he always buys a hot dog. Jabba has identified that the hot dog stand at (4, 3) serves bad hot dogs. He has also identified that both the pretzel stand AND hot dog stand at (7, 8) serve bad pretzels and hot dogs, respectively. He is determined to avoid all three of these stands while still getting his requisite 22 treats on his way to work. How many different paths can Jabba take to work?

Solution

First, we know that all of Jabba's possible walking paths are strings of length 22 consisting of 12 Es and 10 Ns, where an E represents walking east and an N represents walking N. We can use the information in the question to count only the strings that adhere to the given restrictions from this total set of $\binom{22}{10}$ strings.

First, let's just count the total number of paths that go through point (7, 8), since we know that we simply can't visit this point, since both stands are to be avoided at it.

We can get to (7, 8) in $\binom{15}{7}$ ways, since we must choose 7 of the first 15 moves to go east. We can go from (7, 8) to (12, 10) in $\binom{7}{5}$ ways, since we must choose 5 of the last 7 moves to go east. Thus, there are $\binom{15}{7}\binom{7}{5}$ paths that go through the forbidden street corner.

Now, to let's count the number of paths that arrive at (4, 3) by moving east on the last step before getting to (4, 3): This means that the first 6 letters must be 3 N's and 3 E's, which we can arrange in $\binom{6}{3}$ ways. The 7th letter must be an E. The following 15 letters must be 8 Es and 7 Ns, which can be arranged in $\binom{15}{7}$ ways as well. Thus, there are $\binom{6}{3}\binom{15}{7}$ paths where Jabba gets a bad hot dog at (4, 3).

Finally, there are going to be some paths where Jabba gets a bad hot dog at (4, 3) AND visits the point (7, 8). Let's count how many of these there are:

There are $\binom{6}{3}$ paths which get to (4, 3), moving east on the seventh step. From there, there are $\binom{8}{3}$ paths that move from (4, 3) to (7, 8), since here we have to choose 3 times to go east out of the next 8 moves to get to (7, 8). Finally, from (7, 8), there are $\binom{7}{5}$ to get to (12, 10). It follows that there are $\binom{6}{3}\binom{8}{3}\binom{7}{5}$ paths that both go through (4, 3) moving north to arrive at the point AND go through (7, 8).

Using the Inclusion-Exclusion Principle, it follows that the total number of "good ways" Jabba can get to work is:

$$\binom{22}{10} - \binom{15}{7}\binom{7}{5} - \binom{6}{3}\binom{15}{7} + \binom{6}{3}\binom{8}{3}\binom{7}{5}$$

This is sufficiently complicated so it's best to leave the final answer in this form. (It's unlikely that there will be any dramatic simplification of these terms.) Incidentally, this simplifies to 406,331.

Grading: 3 pts for total (22 choose 10)
3 pts to sub out paths through (7, 8)
3 pts to sub out paths through (4, 3) that move east last
1 pt to adjust for I/E

3) (5 pts) A bag contains 6 blue marbles, 7 green marbles and 8 red marbles. If three marbles are chosen out of the bag (without replacement) without looking, what is the probability that all three marbles are blue? Express your answer as a fraction in lowest terms.

Solution

We can choose the three marbles out of 21 in $\binom{21}{3}$ ways. Of these choices, there are $\binom{6}{3}$ ways to choose only blue marbles. Our desired probability is $\frac{\binom{6}{3}}{\binom{21}{3}} = \frac{6\times5\times4}{21\times20\times19} = \frac{2}{133}$.

Grading: 2 pts for denominator, 2 pts for numerator, 1 pt to get to fraction in lowest terms.

4) (10 pts) There are n Christmas lights in a row. Three of these n lights are randomly selected and turned on. The probability that no two of the lights turned on are next to each other is $\frac{39}{60}$. What is n? (Note: You should get a rather ugly quadratic equation when you work this out. Please show all the work to arrive at the quadratic, but use your calculator to find the roots of it.)

Solution

The sample space is $\binom{n}{3}$, since we can choose 3 lights out of n in that many ways.

Of these choices, we can calculate how many don't have consecutive lights by using the n-3 lights turned off as separators:

__ O __ O __ O … __ O __

The lights that are turned on can be placed in the blanks (There are n-2 of these blanks since there are n-3 Os.). We can place at most one light per blank, so we can do this in $\binom{n-2}{3}$ ways.

Thus the desired probability is $\frac{\binom{n-2}{3}}{\binom{n}{3}} = \frac{(n-2)(n-3)(n-4)}{n(n-1)(n-2)} = \frac{(n-3)(n-4)}{n(n-1)}$

Now, set this equal to $\frac{39}{60}$.

$$\frac{(n-3)(n-4)}{n(n-1)} = \frac{39}{60}$$

$$60(n-3)(n-4) = 39n(n-1)$$

$$60n^2 - 420n + 720 = 39n^2 - 39n$$

$$21n^2 - 381n + 720 = 0$$

$$7n^2 - 127n + 240 = 0$$

$$(7n-15)(n-16) = 0$$

Since n is an integer, the desired answer is **n = 16.**

Grading: 2 pts for sample space, 4 pts for numerator, 3 pts to get to quadratic, 1 pt answer

5) (5 pts) In what state did the chain restaurant California Pizza Kitchen open its first store?

California, give to all who submit something for the section.