COT 3100 Final Exam - Part B - 100 pts (12/8/2022)

Last Name: ____________________ , First Name: ______________________

1) (5 pts) Use the laws of logic to show that the two following logical expressions are equivalent:

$$(1)((\bar{p} \vee s) \wedge (\bar{s} \vee \bar{p})) \vee (q \wedge (q \vee r)) \qquad (2) p \to q$$

Please write each step (including commutative and associative) one by one. More lines have been provided than necessary.

$((\bar{p} \vee s) \wedge (\bar{s} \vee \bar{p})) \vee (q \wedge (q \vee r))$	Given

2) (8 pts) UCF is planting several trees in a row on memory mall. Specifically, they've received 10 palm trees, 6 maple trees and 8 pine trees. For aesthetic reasons, the designers have decided that no two palm trees should be planted adjacent to each other. In how many different orders can the trees be planted in a row, with no two palm trees adjacent to each other. (Please leave your answer in combinations/factorials and put a box around your answer.)

3) (10 pts) Consider arbitrary sets finite sets A, B, C, D, E and F such that

$$(A \times B) \cup (C \times D) \subseteq E \times F$$

(a) Prove, via proof by contradiction that $A \subseteq E \vee B \subseteq F$.
(b) Give a small example where the given information is true but $A \nsubseteq E$.
(c) Give a small example where the given information is true but $B \nsubseteq F$.

4) (10 pts) Find all ordered pairs of integers (x, y) that satisfy the equation $354x + 156y = 78$.

5) (8 pts) What is the sum of the divisors of 162,000? **<u>Please give your answer in prime factorized form.</u>**

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6) (8 pts) Consider arranging the following 15 numbers in a mountain sequence:

1, 2, 3, 3, 5, 6, 7, 8, 9, 10, 11, 13, 14, 14, 15

A mountain sequence is one where the unique maximum number is neither the first nor last number in the sequence, and all the values from the first to that maximum number are in strictly increasing order and all the numbers that follow the maximum number are in strictly decreasing order. How many different mountain sequences can be formed using all 15 of the numbers listed above? (Note: an example of one such valid sequence is 1, 2, 3, 6, 7, 8, 11, 13, 14, 15, 14, 10, 9, 5, 3.) **<u>Please simplify your answer to a single positive integer (such as 723).</u>**

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7) (10 pts) Let the probability of tossing heads on a biased coin be $\frac{2}{3}$. Consider tossing the coin n times. Let A represent the parity of n (so if n is 10 A is even, if n is 13 A is odd). Prove, using induction on n, that the probability of rolling an A number of heads (so if n is even an even number of heads, if n is odd, an odd number of heads) is $\frac{1+(\frac{1}{3})^n}{2}$. (Note: Unfortunately, this result doesn't hold for probabilities other than $\frac{2}{3}$, as I found out playing around with the problem.)

8) (10 pts) Let f(x) = ax + b for positive integers a and b with a ≠ 1. In addition, for any function f, define $f^1(n) = f(n)$, and for k > 1, define $f^k(x) = f(f^{k-1}(x))$. Using induction on n, for all positive integers n, prove that $f^n(x) = a^n x + \left(\frac{a^n - 1}{a - 1}\right) b$.

9) (10 pts) Shelly has 10 different pairs of shoes. She picks eight out of the twenty shoes at random. What is the probability that she picked exactly 3 matching pairs of shoes. Please leave your answer as a fraction with combinations and powers as necessary.

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10) (10 pts) Let R be defined over positive integers: $R = \{(a, b) \mid 1 < \gcd(a, b) < \min(a, b)\}$.

Determine, with proof, whether or not R is (a) reflexive, (b) irreflexive, (c) symmetric, (d) anti-symmetric and (e) transitive. (Note: min(a, b) is the smaller of the two values, a or b.)

11) (10 pts) Given positive integers a and n with a < n and gcd(a, n) = 1, prove that no two items in the set S = $\{ai \mid i \in Z \wedge 1 \leq i \leq n\}$ are equivalent mod n. In your proof you may use the following result: if a | bc and gcd(a, b) = 1, then a | c. Please use proof by contradiction.

12) (1 pt) Out of what material is Frosty the Snowman made? ___________________________