COT 3100 Final Exam - Part A (Recitation Topics) - Solutions

1) (8 pts) Sam is taking a road trip in the mountains. Let r be the speed, in miles her hour, that she would have to travel to arrive at her destination on time. For the first third of the length of the trip, there was a rain storm, so Sam drove at r – 10 miles per hour. She calculated that to make it on time, she would have to drive the remaining two thirds of the length of the trip at r + 20 miles per hour. What is the value of r?

Let the distance of the trip be 3D, then the distance of the first third is D and the last two-thirds is 2D. Then we can represent the time for the first segment of the trip, the second segment of the trip and the whole trip, with the sum of the first two equaling the third. This gives us the equation:

$$\frac{D}{r-10}+\frac{2D}{r+20}=\frac{3D}{r}$$

$$\frac{1}{r-10}+\frac{2}{r+20}=\frac{3}{r}$$

$$\frac{r+20+2(r-10)}{(r-10)(r+20)}=\frac{3}{r}$$

$$\frac{r+20+2r-20}{(r-10)(r+20)}=\frac{3}{r}$$

$$\frac{3r}{(r-10)(r+20)}=\frac{3}{r}$$

$$\frac{r}{(r-10)(r+20)}=\frac{1}{r}$$

$$r^2=r^2+10r-200$$

$$10r=200$$

$$\boldsymbol{r=20}$$

Grading: 1 pt for each expression of time (total 3 pts)
1 pt setting these equal
1 pt common denominator
3 pts algebra, give partial as you see fit

Note: 1/8 for correct answer based on guess and check

2) (4 pts) On January 1, 2022, Jessie set her clock to 2:00 PM exactly when it was 2:00 PM, but her clock runs faster than the actual time. Later that evening (same day) when her clock read 10:00 PM, the actual time was 9:36 PM. Assuming that Jessie's clock continues ticking at this rate and she doesn't change it, on what day in January (give the date) will her clock read the exact correct time of 2:00 PM, but the incorrect day, being one day ahead?

Jessie's clock ran 8 hours = 8 x 60 = 480 minutes when the real time only ran 480 – 24 = 456 minutes. This means that the ratio of Jessie's clock to real time is $\frac{480}{456} = \frac{20}{19}$. This means that Jessie's clock will show 20 days elapsed when the real time is 19 elapsed days. It follows that the answer to the question is 19 real days later or **January 20, 2022 at 2:00 PM**. At this time, Jessie's clock will also read 2:00 PM, but on January 21, 2022.

Grading: 1 pt for 480 minutes on Jessie's clock, 1 pt for 456 minutes of real time, 1 pt to set up ratio, 1 pt to arrive at the correct answer.

3) (4 pts) Determine the value of x which satisfies the following equation:

$$log_4 x + log_8(4x^4) = \frac{68}{3}$$

Please express your answer in the form x = 2^a, for some integer a.

Use a common base of 2:

$$\frac{log_2 x}{log_2 4} + \frac{log_2(4x^4)}{log_2 8} = \frac{68}{3}$$

$$\frac{log_2 x}{2} + \frac{log_2 4 + log_2(x^4)}{3} = \frac{68}{3}$$

$$\frac{log_2 x}{2} + \frac{2 + 4log_2 x}{3} = \frac{68}{3}$$

$$\frac{log_2 x}{2} + \frac{4log_2 x}{3} = \frac{66}{3}$$

$$\frac{3log_2 x}{6} + \frac{8log_2 x}{6} = 22$$

$$\frac{11log_2 x}{6} = 22$$

$$log_2 x = 22 \times \frac{6}{11} = 12$$

It follows that **x = 2^{12}.**

4) (5 pts) Let S be an arithmetic sequence such that the sum of the first 25 terms is 638 and the sum of the next 25 terms is 3138. What is the common difference between terms in the sequence S?

Let the sequence be $a_1, a_2, \ldots, a_{25}, a_{26}, \ldots, a_{50}$ with common diffe Note that for integers $i \geq 1$, $a_{25+i} - a_i = 25d$. Let X be the sum of the first 25 terms. Then we can write the sum of the next 25 terms as follows:

$$\sum_{i=26}^{50} a_i = \sum_{i=1}^{25} (a_i + 25d)$$

$$3138 = \left(\sum_{i=1}^{25} a_i\right) + \sum_{i=1}^{25} 25d$$

$$3138 = 638 + 25(25d)$$

$$2500 = 625d$$

$$\boldsymbol{d = 4}$$

Grading: 2 pts for observation that $a_{26} - a_1 = 25d$, 2 pts for then one way or another recognizing that the difference between the two sums is 625d, 1 pt for the answer

5) (4 pts) Consider taking the base 10 representation of 100! and converting it to base 8. How many 0s would be at the end of this converted value?

8 is 2^3, so if we could find out how many times 2 divides evenly into 100! then we could also figure out how many times 8 divides evenly into 100!, which equals the number of 0s at the end of the 100! when represented in base 8. Use the process taught in class:

2 | 100	Number of times 2 divides into 100! = 50 + 25 + 12 + 6 + 3 + 1 = 97
2 | 50	
2 | 25	It follows that 8 divides evenly into 100! exactly $\left\lfloor \frac{97}{3} \right\rfloor = 32$ times.
2 | 12	
2 | 6	Thus, when converted to base 8, 100! has **32 zeroes** at the end of it.
2 | 3	
2 | 1	

Grading: 1 pt for recognizing that 8 is a power of 2, 2 pts for determining how many times 2 divides evenly into 100!, 1 pt for doing an integer division by 3 and getting the final answer.