

**COT 3100 Exam #2 - Part 1 (Number Theory, Recitation Topics) - 30 pts (10/20/2022)**

**Last Name:** \_\_\_\_\_ , **First Name:** \_\_\_\_\_

**Circle Recitation: 8:30 am 10:30 am 11:30 am 12:30 pm 3:30 pm 4:30 pm 7:30 pm**

1) (12 pts) Find all ordered pairs of integer solutions  $(x, y)$ , to the equation

$$309x + 83y = 12$$

**To receive credit, you must use the process shown in class.**

2) (8 pts) The community of Celebration is unique in that all of its houses are identical. Johnny, Selma and Malik are painting houses in Celebration. It takes Johnny 10 days to paint a house in Celebration by himself, it takes Selma 6 days to paint a house in Celebration by herself, and it takes Malik 4 days to paint a house in Celebration by himself. Assume all 3 paint at a constant rate. How long, **in days**, will it take the three of them working together to paint 31 Celebration houses?

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3) (5 pts) What is the remainder when  $7^{22}$  is divided by 11? Use any method you'd like and clearly show your work.

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4) (5 pts)  $X$  is a 3 digit number, when represented in base 10. If  $X$  is represented in base 39, it ends in a 0. If  $X$  is represented in base 45, it ends in a 0. What is the value of  $X$  (in base 10)?

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**COT 3100 Exam #2 - Part 2 (Induction) - 22 pts (10/20/2022)**

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5) (10 pts) Using induction on  $n$ , prove for all positive integers  $n$  that

$$\sum_{i=1}^n \frac{1}{i(i+1)(i+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

6) (12 pts) Prove, using induction on  $n$ , that for all non-negative integers  $n$  that  $16 \mid (15^n - 7^n + 8n)$ .

**COT 3100 Exam #2 - Part 3 (More Induction) - 23 pts (10/20/2022)**

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7) (10 pts) Prove, using strong induction on  $n$ , that the equation  $4x + 5y + 7z = n$  always has at least one non-negative integer ordered triplet  $(x, y, z)$  as a solution for all integers  $n \geq 7$ . You will have to use **four** base cases.

8) (10 pts) Define a sequence of integers,  $s_0, s_1, s_2, \dots$  as follows:

$$s_0 = 3, s_1 = 25, \text{ for all integers } n \geq 2, s_n = 10s_{n-1} - 25s_{n-2}$$

Prove, using strong induction on  $n$ , for all non-negative integers,  $n$ , that  $s_n = (2n+3)5^n$ . (Note: You will use 2 base cases.)

9) (3 pts) With which famous scientist does the restaurant Einstein Brothers Bagels share a name?

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