COT 3100 Sections 2/201 Exam #1 - Part 1 (Logic) - 25 pts (9/15/2022)

Last Name: _____ , First Name: _____

Circle Recitation: 8:30 am 10:30 am 11:30 am 12:30 pm 3:30 pm 4:30 pm 7:30 pm

1) (6 pts) Fill out the following truth table. Please place a T or F in each empty slot. Any ambiguous letter (according to the grader) will be marked incorrect.

p	q	r	$p \vee q$	$q \vee \bar{r}$	$(p \vee q) \wedge (q \vee \bar{r})$
F	F	F			
F	F	T			
F	T	F			
F	T	T			
T	F	F			
T	F	T			
T	T	F			
T	T	T			

2) (10 pts) Using the laws of logic show that the two following logical expressions are equivalent:

(1) $(p \vee (\bar{p} \wedge q)) \vee (r \wedge (\bar{q} \vee r))$ (2) $p \vee q \vee r$

For the purposes of grading, please do NOT skip steps such as Commutative. (There are 2 of these.) There should be 6 steps total, so 7 lines written for your solution, at a minimum.

Step	Reason

3) (5 pts) Prove the following argument via the Rules of Inference. You may skip the Commutative Step in this problem. (It comes up once.)

$$p \rightarrow q$$
$$r \rightarrow s$$
$$p \vee r$$
$$\bar{s}$$

$$\therefore q$$

Number	Step	Reason
1		
2		
3		
4		
5		
6		
7		
8		

Note: You may not use all the rows in the table.

4) (4 pts) Prove or disprove the following assertion over the universe of real numbers:

$$\exists x \forall y [xy = 1]$$

Please clearly note whether the assertion is true or not, followed a justification of your answer. Most of the points are awarded for the justification.

COT 3100 Sections 2/201 Exam #1 - Part 2 (Sets) - 25 pts (9/15/2022)

Last Name: _____ , First Name: _____

Circle Recitation: 8:30 am 10:30 am 11:30 am 12:30 pm 3:30 pm 4:30 pm 7:30 pm

5) (10 pts) Prove or disprove the following assertion for all sets A, B, C and D. **If you prove, please use direct proof or proof by contradiction, do NOT use a set membership table.**

$$(A \cup B) \times (C \cup D) \subseteq (A \times C) \cup (A \times D) \cup (B \times C) \cup (B \times D)$$

6) (10 pts) Prove or disprove the following assertion for all sets A, B and C. **If you prove, please use direct proof or proof by contradiction, do NOT use a set membership table.**

$$if\ A \subseteq B \cap \bar{C}, then\ (B - A) \cap C = \emptyset$$

7) (5 pts) Let A, B and C be sets such that $|A| = 20, |B| = 30, |C| = 40$ and $|A \cap B| = 10$. What is the maximum possible value of $|A \cup B \cup C|$?

COT 3100 Sections 2/201 Exam #1 - Part 3 (D=rt, logs) - 25 pts (9/15/2022)

Last Name: _____ , First Name: _____

Circle Recitation: 8:30 am 10:30 am 11:30 am 12:30 pm 3:30 pm 4:30 pm 7:30 pm

8) (8 pts) Janna does a hike up and down a mountain. She travels at a rate of 2 miles per hour going up the mountain and 4 miles per hour coming down. The total time of her hike is 7 hours (she's a pretty serious hiker =)) and the distance she traversed going down the mountain is 50% greater than the distance going up the mountain. How long, in miles (distance), was her full hike up and down the mountain?

_____

9) (6 pts) Logarithms were originally invented to help astronomers make multiplication calculations easier. Let a = 2.2×10^8 and b = 3.1×10^9. If $log_{10}2.2 = .3424$ and $log_{10}3.1 = .4914$, what is the value of $log_{10}ab$? (Note: An anti-log table would be used in conjunction with the answer to this question to determine the product ab. Also, these tables were more precise than indicated in this question. The goal of the question is simply to illustrate the original use of logarithms!)

_____

10) (10 pts) Find the value of x which satisfies the following equation:

$$log_9(3x^5) + log_{27}(81x^4) = 21$$

_____

11) (1 pt) Dan Marino's (a former NFL quarterback) 61st birthday is today. After which NFL quarterback is your instructor's email address named after?

_____