Problem 1

Sample Space…There are 9 slots, we have 5 odd numbers, we must choose 5 out of 9 slots to place those odd numbers. So our sample space is $\binom{9}{5}$.

Key Observation, each row must have an odd number of odd numbers. (Same for columns.) From 1 to 9, we have 5 odd integers. We have three rows. The only way that 5 odd numbers can be distributed in 3 rows, so that each row has an odd number of odd numbers is (1, 1, 3), (1, 3, 1) or (3, 1, 1) designation. So, let's take an arbitrary look at one of these choices just for the rows:

X	X	X
		X
		X

Once I choose the column for three, then I have to choose the row for three:

	X	
	X	
	X	

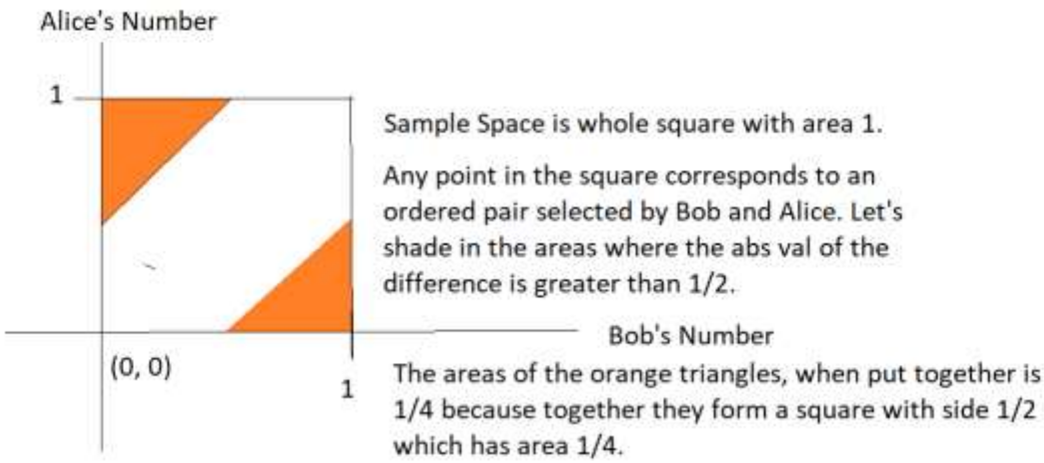
So after choosing column 2 to store three odd numbers, let's say I choose row 1 to store three odds:

X	X	X
	X	
	X	

The number of ways we can satisfy the prompt is # of columns times the # of rows. So we can get an odd sum for all rows and columns in 3 x 3 ways.

Desired probability = $\frac{9}{\binom{9}{5}} = \frac{9(5!4!)}{9!} = \frac{5!4!}{8!} = \frac{4!}{8(7)(6)} = \frac{24}{7(48)} = \mathbf{\frac{1}{14}}$

2) We can visualize our sample space as a 2D grid of the number alice chooses on one axis and the number bob chooses on the other. The solution is simply taking the area of the sections of the square where the situation is satisfied and dividing that by the area of the square:



3) Let N = 5k, k is positive integer.

Let's fine P(N) = P(5k)…

5k + 1 balls, one is red. Let the position values be 0 to 5k, inclusive. If we place the red ball in positions 0 through 2k, inclusive, there are 3k or more green balls to the right. If we place the red ball in positions 3k through 5k, inclusive, there are 3k or more green balls to the left.

Out of the 5k+1 places I could place the red ball, I get a success in any of the 2k+1 + 2k + 1 = 4k+2 locations previously mentioned.

$$\frac{4k+2}{5k+1} < \frac{321}{400}$$

$$400(4k+2) < (5k+1)(321)$$
$$1600k + 800 < 1605k + 321$$
$$479 < 5k$$
$$N > 479$$

Since N is a multiple of 5, N = **480** is the smallest such value.

4) Let p be the probability of landing heads. The probability we get exactly 2 heads is:

$$\binom{4}{2} p^2(1-p)^2 = \frac{1}{6}$$
$$6p^2(1-p)^2 = \frac{1}{6}$$

$$p^2(1-p)^2 = \frac{1}{36}$$

Let X = p(1-p), so we will solve for X first.

$$X^2 = \frac{1}{36}$$

Since X > 0, X = $\frac{1}{6}$. Now, go back to the substitution, so we get:

$$p(1-p) = \frac{1}{6}$$
$$6p(1-p) = 1$$
$$6p^2 - 6p + 1 = 0$$

$$p = \frac{6 \pm \sqrt{36 - 4(6)}}{12} = \frac{6}{12} \pm \frac{\sqrt{12}}{12} = \frac{3 \pm \sqrt{3}}{6}$$

Since p < ½, we have $p = \mathbf{\frac{3-\sqrt{3}}{6}}$**.**