**COT 3100 Recitation #11: Probability 2**
**Quiz and Solutions**

**11/16/2020-11/20/2020**

1) Asha, Bima and Cary take turns aiming at a target. The first person to hit the target wins the game. Asha has a 1/10 chance of hitting the target when she takes a shot, Bima has a 1/20 chance of hitting the target when she takes a shot, and Cary has a 1/5 chance of hitting the target when she takes a shot. Asha goes first, followed by Bima, and then Cary. If all three miss, Asha goes again, and so forth. What is the probability that Asha wins the game?

**Solution**

Let X be the probability that Asha wins the game, when she goes first. Then, the only ways in which Asha can win are:

1) Making the first shot.

2) Having all three miss their shot and then the game starts "again".

This leads to the equation:

$$X = \frac{1}{10} + \frac{9}{10} \times \frac{19}{20} \times \frac{4}{5} \times X$$

$$X = \frac{100}{1000} + \frac{684X}{1000}$$

$$\frac{1000X}{1000} - \frac{684X}{1000} = \frac{100}{1000}$$

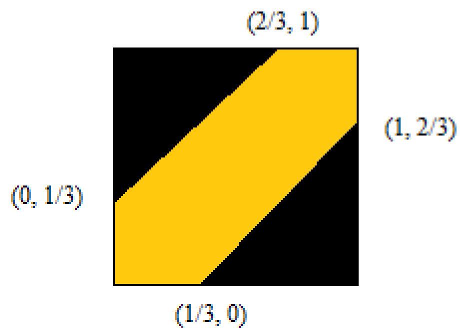
$$\frac{316X}{1000} = \frac{100}{1000}$$

$$X = \frac{100}{316} = \mathbf{\frac{25}{79}}$$

2) Alice and Bob each choose a real number in between 0 and 1, with uniform distribution. What is the probability that the absolute value of the difference between their numbers is less than $\frac{1}{3}$?

**Solution**

We can solve this problem by representing the sample space as a square with side length one, with the x axis representing the value chosen by Alice and the y axis representing the value chosen by Bob. We can draw two lines where the difference between x and y is 1/3. (Specifically, the lines are y - x = 1/3 and x - y = 1/3.) The desired area where the difference between the two is less than 1/3 is in between these two line segments inside the square. To get this area (shaded in gold in the diagram below), subtract out the area in black (two right triangles with legs of length 2/3 which together form a square of side length 2/3) from one to yield 1 - $(2/3)^2$ = 1 - 4/9 = **5/9.** (The probability we desire is the area of the gold divided by the area of the square, but the area of the square is 1!)




3) The digits 1 through 9 are arranged in random order to produce a 9 digit number. What's the probability that the resulting number is divisible by 6?

**Solution**

Since the sum of the digits of each of these 9! numbers is 45, all of the possible numbers that can be formed are divisible by 3. Thus, answering this question is the same as answering what's the probability the randomly created number is divisible by 2 (since 6 = 3 x 2). The probability the last digit is 2, 4, 6 or 8 is precisely 4/9., so the desired probability is **4/9.**

If it's too difficult to see that explanation, consider this alternate one based on two counting problems. The sample space is 9!, the number of permutations of the digits. Of these permutations, we have four choices for the last digit. The remaining 8 digits can be arranged in 8! ways. It follows that the desired probability is $\frac{(4)(8!)}{9!} = \frac{4(8!)}{9(8!)} = \mathbf{\frac{4}{9}}$

4) Four numbers from the set {1, 2, 3, 4, 5, 6, 7, 8, 9} are randomly chosen. What is the probability that the sum of those 4 numbers is odd?

**Solution**

The sample space is the number of ways to choose 4 items out of 9, or $\binom{9}{4} = \frac{9(8)(7)(6)}{1(2)(3)(4)} = 126.$

Now, let's count the number of those combinations whose sum of elements is odd.
To have an odd sum, either 1 or 3 of the values chosen must be odd. There are 5 odd values to choose from, and 4 even values to choose from, so this is, in essence, the lottery problem. We can choose 3 odd values in $\binom{5}{3}$ ways, and choose 1 even value to match with it in $\binom{4}{1}$ ways, Thus, there are $\binom{5}{3}\binom{4}{1} = 10 \times 4 = 40$ combinations with 3 odd numbers and 1 even number.

We can choose 1 odd value in in $\binom{5}{1}$ ways, and choose 3 even values to match with it in $\binom{4}{3}$ ways, Thus, there are $\binom{5}{1}\binom{4}{3} = 5 \times 4 = 20$ combinations with 3 odd numbers and 1 even number.

Thus, there are a total of 60 combinations which have an odd sum out of 126 total. It follows that the desired probability is $\frac{60}{126} = \mathbf{\frac{10}{21}}$**.**

5) A pair of standard 6-sided dice is rolled once. The sum of the numbers rolled determines the perimeter of a square. What is the probability that the numerical value of the area of the square is greater than 5?

**Solution**

We know a square with perimeter 8 has a side length of 2, which is an area of 4. Let's try a square with a perimeter of 9. It has a side length of 9/4, and an area if 81/16 = 5 1/16, which is strictly greater than 5. Thus, we are looking for the probability that we roll a 9, 10, 11 or 12. From class, we had previously worked out these four probabilities to be 4/36, 3/36, 2/36 and 1/36, respectively. Summing we get 10/36 = **5/18** as the desired probability.