

Rec Week 11/9/2020 - Probability #1

Friday, November 6, 2020 4:52 PM

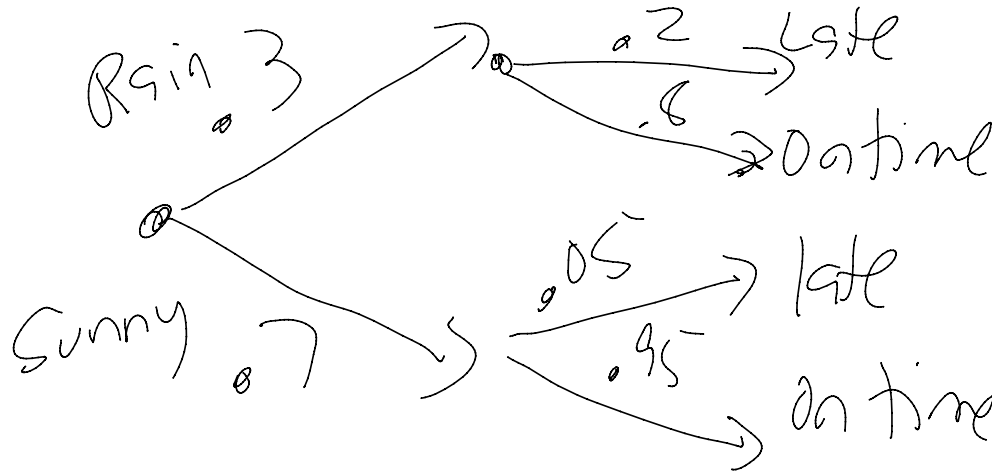
Conditional Probability

Given that it is raining, Sue is late 20%.

Given that it is sunny, Sue is late 5%.

It rains 30% of the time.

One a randomly chosen day, what's the probability Sue is late.



$$p(\text{rain and late}) = .3 \times .2 = 0.06$$

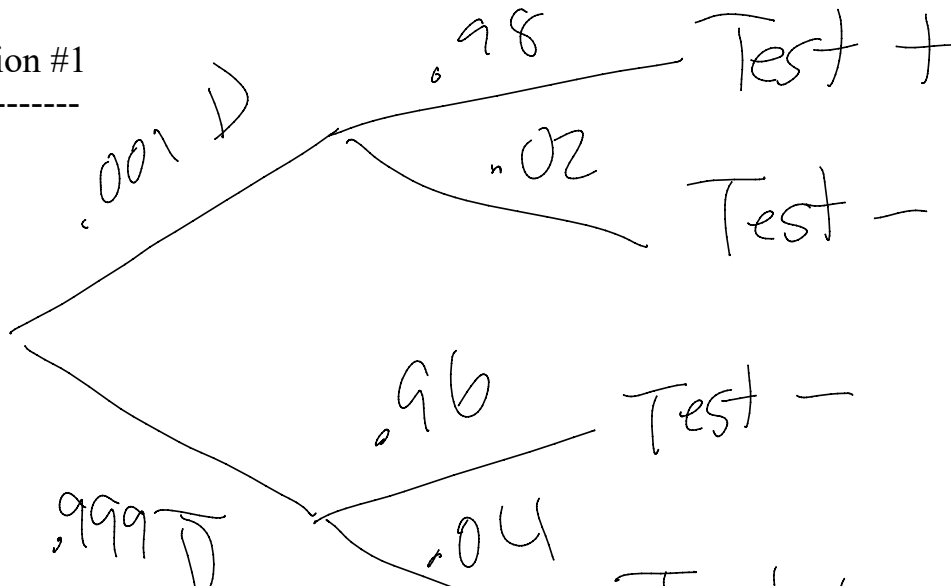
$$p(\text{sunny and late}) = .7 \times .05 = 0.035$$

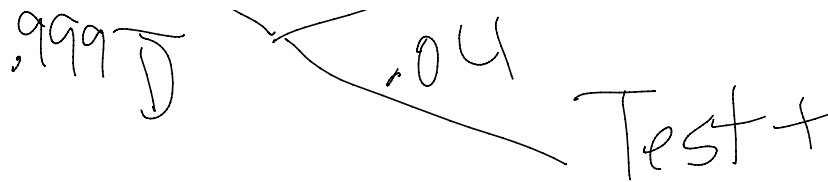
$$p(\text{late}) = p(\text{rain and late}) + p(\text{sunny and late}) = .095$$

I could also ask, given that she was late, what is the probability it was raining day.

$$p(\text{rain} \mid \text{late}) = p(\text{rain and late})/p(\text{late}) = .06/.095 = 0.6316$$

Question #1





$$p(\text{have} \mid \text{tested positive}) = \frac{p(\text{have and test +})}{p(\text{testpositive})}$$

$$= \frac{.00098}{.04094} = 0.02394 \sim 2.4\%$$

$$p(\text{testpositive}) = p(\text{testpositive and have}) + p(\text{testpositive and don't have})$$

$$= .001 \times .98 + .999 \times .04 = 0.04094$$

$$p(\text{testneg}) = 1 - p(\text{testpositive}) = 1 - .04094 = 0.95906$$

$$p(\text{have} \mid \text{test negative}) = \frac{p(\text{have and test -})}{p(\text{testneg})}$$

$$= \frac{.00002}{.95906} = 2.085E-5$$

Binomial Probability Distribution

If we run n trials and our probability of success on a single trial is p , our probability of getting exactly k success is

$$\binom{n}{k} p^k (1-p)^{n-k}$$

Question #2

$$p = .8, n = 15$$

$$\binom{15}{13} \cdot 8^{13} \cdot 2^2 + \binom{15}{14} \cdot 8^{14} \cdot 2^1 + \binom{15}{15} \cdot 8^{15} \cdot 2^0$$

$$\approx 398$$

Definitions of Independent, Mutually Exclusive Events

if A and B are independent, then $p(A) = p(A \mid B)$
 $p(B) = p(B \mid A)$

knowing about A doesn't change the probability of B and vice versa.

if A and B are mutually exclusive, the $p(A \text{ and } B) = 0$.

$$p(A) = 4/9, p(A \text{ and } B) = 2/5, p(A \mid B) = 1/2$$

$$p(A \mid B) = 1/2 = \frac{p(A \text{ and } B)}{p(B)} = \frac{(2/5)}{p(B)}$$

$$p(A) = 1/2, p(A \text{ and } B) = 2/5, p(A | B) = 1/2$$

$$p(A | B) = 1/2 = p(A \text{ and } B)/p(B) = (2/5)/p(B)$$

$$\frac{1}{2} = \frac{2/5}{p(B)} \quad p(B) = \frac{2/5}{1/2} = \boxed{\frac{4}{5}}$$

$$p(B | A) = p(B \text{ and } A)/p(A) = (2/5)/(4/9) = 9/10$$

$$p(B | \text{not } A) = p(B \text{ and not } A)/p(\text{not } A)$$

$$p(\text{not } A) = 1 - p(A) = 1 - 4/9 = 5/9$$

$$p(B) = p(B \text{ and } A) + p(B \text{ and not } A)$$

$$4/5 = 2/5 + p(B \text{ and not } A)$$

$$p(B \text{ and not } A) = 2/5$$

$$p(B | \text{not } A) = p(B \text{ and not } A)/p(\text{not } A)$$

$$= (2/5)/(5/9) = 18/25$$

Question 4

n = 9, p = .2 binomial distribution

$$p(0 \text{ shots}) = \binom{9}{0} \cdot 2^0 \cdot 8^9$$

$$p(1 \text{ shot}) = \binom{9}{1} \cdot 2^1 \cdot 8^8$$

$$p(2 \text{ or more}) = 1 - (.8)^9 - 1.8(.8)^8 \\ \approx .56379$$

$$\text{Expected Value} = \$1000 \times .56379 = \$563.79$$