COT 3100 Recitation #10: Counting 1
Quiz and Solutions

10/26/2020-10/30/2020

1) How many distinct permutations are there of the letters in the word COOKIE?

Solution
There are 6 letters with one repeated, so the answer is 6!/2! = 720/2 = **360.**

2) A Club Membership ID is a 7 digit identification number, where any of the digits is allowed to be 0. However, there are some restrictions on valid ID numbers:

1. The first three digits are consecutive integers, such as 234 or 456.
2. There are no repeated digits in the ID number.

Given these restrictions, how many valid ID numbers are there?

Solution
The first digit can be one of 8 potential values, ranging from 0 to 7, since we need to leave room for the third digit which is the first digit plus 2. The second and third digits can only be 1 possible value, based on the first digit. At this point, 3 out of 10 digits have been used. The fourth digit can be one of 7 choices, the fifth digit can be one of 6 choices, the sixth digit can be one of 5 choices and the seventh digit can be one of 4 choices. It follows that the desired answer is 8 x 7 x 6 x 5 x 4 = 8!/3! = **8!/6**, one of the listed answer choices.

3) A class has 17 girls and 13 boys. The class will nominate a committee of 3 girls and 2 boys to represent the class. How many different possible committees can be nominated?

Solution
We can choose any 3 girls out of 17 in ${}_{17}C_3$ ways.
We can choose any 2 boys out of 13 in ${}_{15}C_2$ ways.
We can pair up any choice of girls with any choice of boys, like an ordered pair, to create a full committee.
Thus, to get the total number of possible committees, we multiply these two numbers to get a possible **$({}_{17}C_3)({}_{15}C_2)$** committees.

4) Sarah is arranging 10 roses and 4 tulips in a single row. She doesn't want any of the tulips next to each other. In how many ways can she arrange her flowers. (Note: 2 roses are indistinguishable and 2 tulips are indistinguishable.)

Solution
Let the roses be separators:

__ R __ R __ R __ R __ R __ R __ R __ R __ R __ R __

There are 11 gaps where the tulips can be placed. For each gap, we either select it or don't select it. Thus, we must select precisely 4 out of 11 of these gaps to place the tulips. We can do this in $_{11}C_4$ ways.

5) Binary Billy starts writing numbers in binary, starting with 1. He starts writing, "1, 10, 11, 100, 101, 110, 111, …" When Binary Billy is writing his 1000th bit, what number (convert to decimal) is he writing out?

Solution
There is one number with 1 bit.
There are two numbers with 2 bits.
There are four numbers with 3 bits.
There are eight numbers with 4 bits.
In general, there are 2^{n-1} numbers with n bits, since the first bit has to be one and for the rest of the n-1 bits we have two choices. Start adding up the number of bits written, stopping when it looks like an individual product is in the neighborhood of 1000:

1 x 1 + 2 x 2 + 4 x 3 + 8 x 4 + 16 x 5 + 32 x 6 + 64 x 7 =

1 + 4 + 12 + 32 + 80 + 192 + 448 =

769

So, after writing 1111111, Billy has 1000 - 769 = 231 more bits to write. Each of these subsequent numbers will have 8 bits.

Thus, he completes writing $\left\lfloor \frac{231}{8} \right\rfloor = 28$ numbers. These 28 numbers, in decimal, are 128, the first number with 8 bits through, 155, inclusive. (Specifically, after writing 155, Billy has written 769 + 8*28 = 993 bits. He writes his 1000th bit writing the decimal number 156, which, when he writes it in binary is 10011100. Thus, the 1000th bit he writes is 0 (highlighted), as part of the number that is the binary representation of **156.**