COT 3100 Recitation #9: Random Algebra Problems
Quiz and Solutions

10/19/2020-10/23/2020

1) Sasha and her mother's ages add up to 100 currently. Thirty years ago, Sasha's mother was 7 times as old as Sasha. In five years, what birthday will Sasha's mother celebrate?

Solution
Let s be Sasha's age and m be her mother's age. Using the given information, we have:

s + m = 100, so m = 100 - s
7(s-30) = m-30
7s - 210 = (100-s) - 30
7s - 210 = 70 - s
8s = 280
s = 35, so m = 100 - 35 = 65

In five years, Sasha's mother will be 65 + 5 = **70 years old.**

2) At noon, Dwayne synchronizes his watch to the official NIST time. At 5 pm of the same day (official NIST time), Dwayne checks his watch and it reads 5:15 pm, which means his clock is ticking faster than the real time. Assuming that the rate of his clock's ticking is constant and continues, when Dwayne's watch reads 12:15 am (so his watch has gone past midnight by 15 minutes), what is the official NIST time?

Solution
In 5 hours = 300 minutes of real time, Dwayne's watch registers 315 minutes, so Dwayne's watch ticks $\frac{315}{300} = \frac{21}{20}$ of the rate of the real time. Let x equal the actual number of minutes that has elapsed since noon when Dwayne's watch reads 12:15 am. At this time, Dwayne's watch has gone through 12 x 60 + 15 = 735 minutes. Thus, we can set up the following equation to solve for x, the number of actual elapsed minutes:

$$\frac{21}{20} = \frac{735}{x}$$

$$x = \frac{735 \times 20}{21} = 35 \times 20 = 700$$

Thus, 700 real minutes have elapsed since noon, which is 11 hours (660 minutes) and 40 minutes. Thus, the current real time is **11:40 pm.**

3) In a mixture of acid and water, after 2 ounces of acid are added, the overall mixture is 50% acid. Subsequently, 8 ounces of water are added. This final mixture is 30% acid. What percentage of the original mixture was acid?

Solution
Let a be the number of ounces of acid in the original mixture and t be the total number of ounces of the original mixture. (Thus, t-a represents the amount of water in the original mixture.)

Using the given information, we have $\frac{a+2}{t+2} = \frac{1}{2}$, and $\frac{a+2}{t+10} = \frac{3}{10}$.

Using the first equation, we have $t + 2 = 2(a + 2) = 2a + 4$, it follows that $t = 2a + 2$.

Substitute in the second equation:

$$\frac{a+2}{2a+2+10} = \frac{3}{10}$$

$$10(a + 2) = 3(2a + 12)$$

$$10a + 20 = 6a + 36$$
$$4a = 16$$
$$a = 4$$

It follows that $t = 2(4) + 2 = 10$, and the original mixture was $\frac{4}{10} \times 100\% = \mathbf{40}\%$ **acid.**

4) Jerry writes down a two digit number, X, with the ten's digit less than the one's digit. (Note: the ten's digit is not 0.) Jerry swaps the two digits to create a larger two digit number, Y. Jerry calculates Y - X and then sums the digits of this new number. What is the sum of the digits of the number Y-X?

Solution
Let the ten's digit of X be a and the one's digit of X be b. Thus, X = 10a + b. It follows that Y = 10b + a. Subtracting, we have:

Y - X = (10b + a) - (10a + b) = 9b - 9a = 9(b-a).

Since a > 0 and b > a, b-a must be in between 1 and 8. Thus, this difference must be 9, 18, 27, 36, 45, 54, 63 or 72. In all cases, the sum of the digits is **9.**

A more elegant way to show this is to note that the difference is a multiple of 9. Let c be the ten's digit of the difference (possibly 0) and d be the one's digit of the difference. Then, we have

Y - X = 10c + d ≡ c + d (mod 9)

Thus, the value of the number is equivalent to the sum of its digits mod 9. We know that the value of the number is 9(b - a) ≡ 0 (mod 9). Thus, the sum of digits desired is divisible by 9. Since that sum can't be 0 and the smallest multiple of 9 with a digit sum greater than 9 is 99, it follows that the corresponding digit sum must be 9.

5) One orange costs five cents more than one apple. If the total cost of four apples and five oranges is $2.05, how much does an apple cost?

Solution

Let a be the cost of an apple, in cents. Then a+5 is the cost of an orange, in cents. Then, using the given information, we have:

$4a + 5(a+5) = 205$

$4a + 5a + 25 = 205$

$9a = 180$

$a = 20$

Thus, an apple costs **20 cents.**