Fall 2019 COT 3100 Exam #2 (10/24/2019) (Note: Out of 100 points) - Pages 1, 2

Last Name: _____ , First Name : _____

Lab Section: 18(R9) 19(R10) 20(R11) 21(T2) 22(T3) 23(T4) 24(T5)

1) (10 pts) Eileen rides her bike a distance of D miles at a rate of r miles per hour from home to the bus stop. She then takes the bus for a distance of 5D miles traveling at a rate of 4r miles per hour to arrive at work. Assume that the bus starts immediately after Eileen arrives to the bus stop. If the average speed for Eileen's trip from home to work is 32 miles per hour, at what rate, in miles per hour, does she ride her bike for that portion of the trip?

2) (12 pts) Consider a particle on the Cartesian plane that starts at point (0, 0) at time t=0, initially headed in the direction of the positive y-axis. At time t, the particle will move t+1 units in the direction that it is headed, with the first movement starting at t=0. Thus, at t=1, the particle will end up at (0,1). After each move, the particle will turn left 90 degrees, instantaneously. Thus, at t=2, the particle will end up at (-2, 1), since it's headed in the direction of the negative x-axis and travels 2 units in that direction. Using induction on n, prove for all non-negative integers n, that at time t = 4n, the particle will be located at (2n, -2n), heading in the positive y-axis.

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3) (15 pts) Find all integer solutions to the equation $258x + 204y = 42$.

4) (10 pts) Let x and y be integers such that $19 \mid (5x - y)$. Prove that $19 \mid (x+15y)$.

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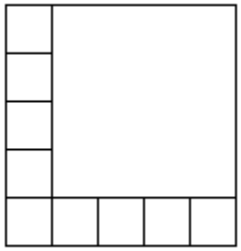
5) (15 pts) Using strong induction on n with three base cases, prove that a square can be partitioned into n squares, for all positive integers $n \geq 6$. (Note: To partition a square, you must draw some line segments dividing the square so that all of the separate pieces are non-intersecting, cover the whole square, and are all squares themselves. The picture below shows a square partitioned into 10 squares. Specifically in this picture, if the original square has side length s, 9 of the squares have side length $\frac{s}{5}$, and the tenth square in the upper right hand corner has side length $\frac{4s}{5}$.) Your proof should primarily have pictures accompanied with the specific dimensions of each of the smaller squares in terms of the square to be partitioned.



6) (14 pts) Let p be an positive integer (greater than 1) such that $2^p - 1$ is a prime number. Let $n = 2^{p-1}(2^p-1)$. Determine the sum of divisors of n, in terms of either n, p or both. **(Note: Full credit is given only for the most simple form of the final answer.)**

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7) (12 pts) Using induction on n, prove for all non-negative integers n that $\sum_{i=0}^{n} \frac{i}{2^i} = 2 - \frac{n+2}{2^n}$.

8) (10 pts) On Bobby's first day of work, he earns $100. On every subsequent day, he earns $4 more than the previous day. On Selena's first day of work, she earns $10. On every subsequent day, she earns $6 more than the previous day. Both start work on the same day. After n days of both working, they compare their ***total*** earnings to that point in time and realize that Selena has earned ***strictly more money*** in total than Bobby. What is the minimum possible value of n?

9) (2 pts) By what acronym is the National Basketball Association typically referred? _____