

# Average Problems, Median, Range

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Definition of Average (Mean) of a list of numbers is simply the sum of those numbers divided by the number of numbers.

List: 3, 5, 5, 4, 6, 9, 2, 8

$$\text{Average} = (3+5+5+4+6+9+2+8)/8 = 5.25$$

$$\text{Average} = \text{Sum}/(\# \text{ terms})$$

$$\text{Sum} = \text{Average} * (\# \text{ terms})$$

Special Case

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Average of consecutive integers:

3, 4, 5, 6, 7, 8, 9 (odd # of integers there are 7)

$m-3, m-2, m-1, m, m+1, m+2, m+3$

Average of 7 consecutive integers is simply sum of those values above, divided by 7 is equal to  $7m/7 = m$ .

11, 12, 13, 14 (we can call the 2 middle values  $m-.5, m+.5$ )

$m-1.5, m-.5, m+.5, m+1.5$

same cancellation idea. Sum is  $4m$ , where  $m$  is the average of the two middle values.

Median - middle value...in an odd # list of terms, it's the actual middle value. In a even # list of terms, it's the average of the middle two values.

Here is a list of values: 4, 9, 6, 4, x. What are the possible values of the median, given that x is some integer.

4, 4, 6, 9, x

if  $x < 4$ , median is 4

if  $x \geq 4$  and  $x \leq 6$ , median x

if  $x > 6$ , median is still 6

Try same problem even # of values: 9, 8, 3, 9, 11, x

3, 8, 9, 9, 11, x

if  $x \leq 8$ , median is 8.5

if  $x \geq 9$  median is still 9

if x doesn't have to be an integer, then if

$x > 8$  and  $x < 9$ , median is  $(x+9)/2$ .

Range = max - min

Sometimes the data in average problems will be given with a frequency table

Example: survey we ask people # of siblings they have.

15 people have 0 siblings

13 people have 1 sibling

12 people have 2 siblings

5 people have 3 siblings

2 people have 4 siblings

real data:

0,0,0,0...0, 1,1,1,...,1,2,2,2,...,2,3,3,3,3,3,4,4

$n$  (number of people) =  $15 + 13 + 12 + 5 + 2 = 47$

sum of the data (# of siblings) =  $15*0 + 13*1 + 12*2 + 5*3 + 2*4 = 60$

Avg # of siblings =  $60/47 = 1.2766$

- 1) 5 consec integers starting with a have average b:  $a, a+1, a+2, a+3, a+4$ . Average is the middle value which is  $a+2 = b$ . The 5 consec integer starting with b are  $b, b+1, b+2, b+3, b+4$ . Average is  $b+2 = a+2+2 = \mathbf{a+4}$ .
- 2) list of 50 #s has an average of 38. If we remove 45,55 from the list what is the new average of the remaining 48 #s.

$$\text{Sum of 50 \#s} = 50 \times 38 = 1,900$$

$$\text{Sum of 48 \#s} = 1900 - 45 - 55 = 1800$$

$$\text{Average of those 48 \#s} = 1800/48 = 600/16 = 300/8 = \mathbf{75/2}$$

- 3) Ratio W:M is 11:10. Avg age women = 34, avg age men = 32  
let there be  $11x$  women,  $10x$  men, for some integer  $x$ .

$$\text{Sum of ages of the women} = (11x)34 = 374x$$

$$\text{Sum of the ages of men} = (10x)32 = 320x$$

$$\text{Sum of all ages} = 374x + 320x = 694x$$

$$\text{Total \# of people} = 11x + 10x = 21x$$

$$\text{Average age of all} = 694x/21x = 694/21 = \mathbf{33 \frac{1}{21}}$$

- 4) Let  $X$  be the sum of the # of problems solved by people who solve in between 4 and 12 problems in total. Let  $Y$  equal the total number of people who solved in between 4 and 12 problems in total.

$$\text{\# of people who solved 3 or more} = Y + 23 + 5 + 2 + 1 = Y + 31$$

$$\text{sum of scores people solved 3 or more} = X + 3*23 + 13*5 + 14*2 + 15*1 = X + 177$$

$$\text{Equation} = (X+177)/(Y+31) = 6 \rightarrow X+177 = 6Y + 186$$

$$\text{\# of people who solved 12 or fewer} = Y + 23 + 7 + 5 + 9 = Y + 44$$

$$\text{Sum of people who solved 12 or fewer} = X + 3*23 + 2*7 + 5*1 = X + 88$$

$$\text{Equation} = (X+88)/(Y+44) = 5 \rightarrow X + 88 = 5Y + 220$$

$$X + 177 = 6Y + 186 \rightarrow X = 6Y + 9$$

$$X + 88 = 5Y + 220 \rightarrow X = 5Y + 132$$

$$6Y + 9 = 5Y + 132$$

$$Y = 132 - 9 = 123$$

$$X = 6Y + 9 = 6*123 + 9 = 747$$

$$\begin{aligned} \text{Total sum of all} &= 747 + 13*5 + 14*2 + 15*1 + 3*23 + 2*7 + 5*1 \\ &= 747 + 65 + 28 + 15 + 69 + 14 + 5 = \mathbf{943} \end{aligned}$$