COT 3100 Recitation #7: Problems Dealing with Mean and Mode Solution
10/5/2020-10/9/2020

<u>Problem Solved in Recording</u>

1) Five positive consecutive integers starting with a have average b. What is the average of 5 consecutive integers that start with b, in terms of a?

<u>Solution</u>

The five consecutive integers starting with a are a, a+1, a+2, a+3, and a+4. Their average is a+2. (The average of an arithmetic series with an odd number of terms is always the middle term.) Thus, $b = a + 2$. Similarly, the average of 5 consecutive integers starting with b is b+2 = **<u>a+4.</u>**

2) A list of 50 numbers has an average of 38. If we remove the values 45 and 55 from the list, what is the new average of the remaining 48 numbers?

<u>Solution</u>

The sum of the numbers is 50 x 38 = 1900. If we remove the two values from the list, the resulting sum is 1800 and the number of values is 48. The corresponding average is 1800/48 = 37.5.

3) In a certain population the ratio of the number of women to the number of men is 11 to 10. If the average age of the women is 34 and the average age of the men is 32, then what is the average age of the population?

<u>Solution</u>

Let 11x equal the number of women and 10x equal the number of men. There are 21x people in the population. If the average age of the women is 34, the sum of their ages is 34(11x) = 374x. If the average age of the men is 32, the sum of their ages is 32(10x) = 320x. The sum of everyone's ages is 374x + 320x = 694x. Thus, the average age of the population is $\frac{694x}{21x} = 33\frac{1}{21}$.

4) The table below shows the number of contestants who solved n problems correct on a previous mathematics exam which contained 15 questions:

n	0	1	2	3	…	13	14	15
number of contestants solving n problems	9	5	7	23		5	2	1

In addition, the following facts are known about the exam:

Those who answered 3 or more questions correctly averaged solving 6 questions each.
Those who answered 12 of fewer questions correctly averaged solving 5 questions each.

What was the sum of the number of problems solved by each of the contestants?

Solution
Let x be the number of people who scored in between 4 and 12 and let y be the sum of their scores. Using the given information, about those who answered 3 or more questions we have:

(23*3 + y + 13*5 + 14*2 + 15*1)/(23+x+5+2+1) = 6 so y + 177 = 186 + 6x, and y = 6x+9

and using the given information about how answered 12 or fewer we have:

(0*9 + 1*5 + 2*7 + 3*23 + y)/(9+5+7+23+x) = 5, so y + 88 = 5x + 220, and y = 5x + 132

Set these two expressions for y equal to one another:

6x + 9 = 5x + 132
x = 123

Now, solve for y = 6(123) + 9 = 747.

Finally, add to the 747, the total number of questions solved by those who solve 3 or fewer or more than 12: 747 + 1*5 + 2*7 + 3*23 + 13*5 + 14*2 + 15*1 = **943.**

Problems for Recitation

1) The average value of all the pennies, nickels, dimes and quarters in Paula's purse is 20 cents. If she had one more quarter, the average value would be 21 cents. How many dimes does she have in her purse?

Solution
Let p be the number of pennies, n the number of nickels, d the number of dimes and q the number of quarters. Then we have:

(p+5n+10d+25q)/(p+n+d+q) = 20, so p + 5n + 10d + 25q = 20(p+n+d+q)

If we add an extra quarter, we have:

(p+5n+10d+25q+25)/(p+n+d+q+1) = 21, so p + 5n + 10d + 25q +25 = 21(p+n+d+q) + 21

Substitute p + 5n + 10d + 25q = 20(p+n+d+q) into the second equation:

20(p+n+d+q) + 25 = 21(p+n+d+q) + 21
p+n+d+q = 4

Thus, there are four coins total in Paula's purse. If she only had 2 quarters, the maximum value would be 2(25) + 2(10) = 70 cents. But, she has 80 cents (since the average of her 4 coins is 20 cents). Thus, she has to have at least 3 quarters. This means the last coin is a nickel, which is the only way to get 80 cents with four coins, of which, 3 are quarters. Thus, she has **0 dimes** in her purse.

2) The 36 distinct integers from -15 to 20, inclusive fill a 6 x 6 grid such that each row and column sum to the same value. What is this row/column sum?

Solution

The sum of the 36 integers is -15 - 14 - 13 … + 14 + 15 + 16 + 17 + 18 + 19 + 20 = 16 + 17 + 18 + 19 + 20 = 90. Since the sum of each of the six rows is the same, let R equal the sum of a row. Then we have 6R = 90, so **R = 15.**

3) For each positive integer n, the mean of the first n terms of the sequence is n. What is the 2020^{th} term of the sequence?

Solution

If the average of the first n terms is n, then the sum of the first n terms is n^2. It follows that the sum of the first 2020 terms is 2020^2 and the sum of the first 2019 terms is 2019^2. The 2020^{th} term is the difference between these two, so we get $2020^2 - 2019^2$ = (2020-2019)(2020+2019) = **4039.**

4) For positive integers m and n such that m + 10 < n + 1, both the mean and the median of the set {m, m+4, m+10, n+1, n+2, 2n} are equal to n. What is m + n?

Solution

This means the values are in sorted order and that the sum of the values is 6n. In addition, we know that m+10 = n-1 since the median is n. Thus, we have m = n - 11. In terms of n the 6 values are:

n - 11, n - 7, n - 1, n+1, n+2, 2n

The sum of these values is 7n - 16 = 6n. It follows that n = 16. Plugging in for n in m = n - 11, we get m = 5. The sum m+n = **21.** We double check the solution by producing the set:

{5, 9, 15, 17, 18, 32}

This set has a median of 16 and an average of 16, verifying the solution.