COT 3100 Recitation #7: Problems Dealing with Mean and Mode
Quiz and Solutions

10/5/2020-10/9/2020

1) The mean, median and mode of the 7 data values 60, 100, x, 40, 50, 200, 90 are all equal to x. What is the value of x?

Solution
First, sort the numbers without x:

40, 50, 60, 90, 100, 200

The sum of these numbers is 540. After we add x, the sum of the 7 values is 540 + x. The average of these values is simply (540+x)/7, but we are also told that the average of the values is x, so we have the equation:

$(540+x)/7 = x$
$540 + x = 7x$
$6x = 540$
$x = 90$

The list of numbers is 40, 50, 60, 90, 90, 100, 200, which does indeed have a mean, median and mode of **90.**

2) Mr. Patrick teaches math to 15 students. He was grading tests and found that when he graded everyone but Patrick's the average was 80. After he graded Patrick's test, the class average became 81. What was payton's score on the test? (2015AMC12A-3)

Solution
The average of the first 14 tests was 80, so the sum of those 14 test scores is 14 x 80 = 1120. If the average of the 15 tests was 81, then the sum of all 15 test scores is 15 x 81 = 1215. It follows that Patrick's score was 1215 - 1120 = **95.**

3) In an election between candidate X and candidate Y, 100,000 voters have voted early and candidate X has received 48% of the vote. (Note: for the purposes of this question, every voter must vote for exactly one candidate between candidate X and candidate Y.) It is known that 50,000 voters will show up on election day. What percentage of those voters have to vote for candidate X so that both candidates receive the same number of votes?

Solution
Candidate X has 48,000 early votes. Since there are 150,000 total votes casted, she needs half of that many, or 75,000. Thus, she needs 75,000 - 48,000 = 27,000 votes on election day to have the

same number of votes as Candidate Y. As a percentage, 27,000 out of 50,000 is **54%.** (Formally, set up a fraction of 27,000/50,000 = x/100 and solve for x.)

4) In a group of 11 people, the average height of the 10 shortest people is 64 inches and the average height of the 10 tallest people is 66 inches. What is the range of the heights of the people in the group? (Recall that range is defined as the maximum value in the data minus the minimum value in the data.)

Solution

Let the heights of the 11 people, in order be h_1, h_2, …, h_{11}. Since the average of the first 10 terms is 64, we have

$h_1 + h_2 + h_3 + \ldots + h_{10} = 64 \times 10 = 640$.

Similarly, since the average of the last 10 terms is 66, we have:

$h_2 + h_3 + h_4 + \ldots + h_{11} = 66 \times 10 = 660$.

Subtract the bottom equation from the top and you get:

$h_{11} - h_1 =$ **20.**

Notice that this expression is precisely the range - the maximum value minus the minimum value. Thus the range is **20 inches.**

5) For each positive integer n, the mean of the first n terms of the sequence is n^2. In terms of n, what is the n^{th} term of the sequence?

Solution

This means that the sum of the first n terms is $n \times n^2 = n^3$. To get the n^{th} term, simply subtract the sum of the first n-1 terms from the sum of the first n terms. Thus, the desired answer is:

$n^3 - (n-1)^3 = n^3 - (n^3 - 3n^2 + 3n - 1) =$ **$3n^2 - 3n + 1$.**