Factorizations

Monday, September 28, 2020 10:33 PM

$x^2 - y^2 = (x + y)(x - y)$

What is 21 x 29?

Halfway in between is 25…

$(25 - 4) \times (25 + 4) = 25^2 - 4^2 = 625 - 16 = 609$

One key is that you can have any expression for x and any expression for y.

$(a + b)^2 - (c+d)^2 = (a + b + c + d)(a + b - c - d)$, for example.

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

$x^n - y^n = (x - y)(x^{n-1} + x^{n-2}y + x^{n-3}y^2 + \ldots + y^{n-1})$

In its most common use, y is usually 1…

The product of two boys's ages is 27, and both are children of different ages. How old are both of the boys?

$xy = 27$

1 x 27 (adult)
3 x 9, they must be 3 and 9, respectively.

With most numbers, the factors come in pairs:

24 = 1 x 24
 2 x 12
 3 x 8
 4 x 6

The exception perfect squares

36 = 1 x 36
 2 x 18
 3 x 12
 4 x 9
 6 x 6

All positive integers have an even number of divisors except for perfects squares and one way to show it is this pairing property.

For our matching pairs (except the square root), one is less than the square root of the number and one is greater than the square root of the number.

$n = 2^3 5^7 7^3 11^1$, any divisor of this number will be of the form

$2^a 5^b 7^c 11^d$, where $0 <= a <= 3$, $0 <= b <= 7$, $0 <= c <= 3$, $0 <= d <= 1$,

the values of a,b,c, and d are independent, so we can pair any a up with any b and so forth.

There are 4 possible values for a, 8 possible values for b, 4 possible values for c and 2 possible values for d. The total number of divisors of n is 4 x 8 x 4 x 2 = 256

divisors = product of each exponent plus 1.

To have an odd number of divisors, each term must be odd, but if each term is odd, then each exponent is even. Thus, only perfect squares have an odd number of divisors.

1) $x^2 - ax + 2a = 0$

$$x = \frac{a \pm \sqrt{a^2 - 8a}}{2}$$

$a = -1$

$a^2 - 8a + 16 = b^2 + 16 \quad b \in \mathbb{Z}$

$a = 0, 8, -1, 9$

$(a-4)^2 = b^2 + 16$

Sum = 16

$(a-4)^2 - b^2 = 16$

(Sum 16

$$(a-4+b)(a-4-b)=16$$

$\cancel{a=12.5}$

$a=9,\ b=3$

$x^2-9a+18$

$(x-3)(x-6)$

16	1	
8	2	
4	4	$\longrightarrow a=0,8$
-2	-8	$a=-1$

2) $1260x = N^3$

$$2\cdot 5\cdot 2\cdot 3^2\cdot 7x = N^3$$

$$2^2\cdot 3^2\cdot 5^1\cdot 7^1 x = 2^3\cdot 3^3\cdot 5^3\cdot 7^3$$

$$x = 2^1\cdot 3^1\cdot 5^2\cdot 7^2$$

$$7350$$

3) $10!$ in base 12

$$10! = 12^n x,\ 12 \nmid x.$$

$$=2\cdot 3\cdot 2^2\cdot 5^1\cdot 2\cdot 3\cdot 7\cdot 2^3\cdot 3^2\cdot 2\cdot 5$$

$= $ [illegible]

$= 2^8 \cdot 3^4 \cdot 5^2 \cdot 7^1$

$12^n = 2^{2n} \cdot 3^n$

$\boxed{n = 4 \text{ max}}$

4) $A + M + C = 12$

$\max(AMC + AM + AC + MC)$

$\boxed{AMC + AM + AC + MC + A + M + C + 1}$

$= (A+1)(M+1)(C+1)$

$(AMC + AM + AC + MC) + 13$

to maximize just maximize

just set $A+1 = M+1 = C+1$ to max product

$A = M = C = 4$

$(A+1)(M+1)(C+1) = 125$ (max)

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Ans: $125 - 13 = \boxed{112}$