COT 3100 Recitation #6: Use of Factorization of Integers, Algebraic Expressions
Quiz and Solutions

9/28/2020 - 10/2/2020

1) Bob is Jill's older brother. Both are children and the product of their ages is 44. How old is Jill?

Solution
44 can be represented as the product of two distinct integers as follows: 1 x 44, 2 x 22 and 4 x 11. Of these options, only the last option contains ages of children. Since Bob is older, it follows that Jill is **4.**

2) What is the largest prime factor of 400! ?

Solution
400! = 1 x 2 x 3 … x … x 400, so to prime factorize this number, you just have to prime factorize each term. 400 isn't prime (divisible by 2). 399 isn't prime (divisible by 3). 398 isn't prime (divisible by 2), 397 is prime. Here are the remainders when 397 is divided by each prime upto 20:

$397 \equiv 1 \pmod{2}$
$397 \equiv 1 \pmod{3}$
$397 \equiv 2 \pmod{5}$
$397 \equiv 5 \pmod{7}$
$397 \equiv 1 \pmod{11}$
$397 \equiv 7 \pmod{13}$
$397 \equiv 6 \pmod{17}$
$397 \equiv 17 \pmod{19}$

It follows that **397** is prime, and all of the other prime factors of 400! must be smaller.

3) How many divisors does 250,000 have?

Solution
$250000 = 25 \times 10^4 = 5^2 \times (2 \times 5)^4 = 5^2 \times 2^4 \times 5^4 = 2^4 \times 5^6$

Thus, 250000 has (4+1)(6+1) = **35 divisors.**

4) Both roots of the quadratic equation $x^2 - 103x + k = 0$ are prime numbers. What is the value of k?

Solution
The sum of the roots is 103. Two integers can only sum to be odd if one is even and one is odd. 2 is the only prime number, so one of the two roots is 2. The other must be 101. Since k is the product of the roots, k = 2 x 101 = **202.**

5) $2^{36} - 1$ has only one prime divisor in between 40 and 100. What is that prime divisor?

Solution

$2^{36} - 1 = (2^{18} - 1)(2^{18} + 1)$

$= (2^9 - 1)(2^9 + 1)(2^6 + 1)(2^{12} - 2^6 + 1)$

$= (2^3 - 1)(2^6 + 2^3 + 1)(2^3 + 1)(2^6 - 2^3 + 1)(2^{12} - 2^6 + 1)$

$= 7 \times 73 \times 9 \times 57 \times (2^{12} - 2^6 + 1)$

$= 7 \times 73 \times 9 \times 3 \times 19 \times (2^{12} - 2^6 + 1)$

One of the listed numbers, **73**, is prime. Thus, using the given information, we can ascertain that when the number is fully broken down, no other prime divisors are in between 40 and 100.

Incidentally, $2^{36} - 1 = 3^3 \times 5 \times 7 \times 13 \times 19 \times 37 \times 73 \times 109$