A/G Series Problems

Monday, September 21, 2020 6:14 PM

1) $a_4 = 13$, $a_{10} = 31$, what is the sum of the 1st 100 terms?
Let d be the common difference of the series. then,
$a_{10} = a_4 + (10 - 4)d$
$31 = 13 + 6d$
$18 = 6d$
$d = 3$

$S = \left(\frac{a_1 + a_{100}}{2}\right) \cdot 100$

$a_{100} = a_4 + (100 - 4)3$
$= 13 + 96 \times 3$
$= 301$

$a_1 = a_4 + (1 - 4)3$
$= 13 - 9 = 4$

$= \left(\frac{4 + 301}{2}\right) \cdot 100$

$= 50 \times 305$

$= \boxed{15{,}250}$

2) Let n be the number of terms in the sequence, let a_i be the ith term of the sequence and let S_n represent the sum of the first n terms of the sequence. We have:
$a_1 = 2$, $a_n = 29$, $S_n = 155$

$155 = ((2 + 29)/2)*n$
$310 = 31n$
$n = 10$

$a_{10} = 29 = a_1 + 9d = 2 + 9d$
$27 = 9d$
$d = 3$

3) Use the same naming conventions as the last problem, so we have:
$S_{98} = 137$, $d = 1$

$a_1 = a_2 - 1$

$137 = a_1 + a_3 + a_5 + \cdots a_{97}$
$a_2 + a_4 + a_6 + \cdots + a_{98}$

$a_2 + a_4 + a_6 \quad + a_{98}$?

$137 = (a_2 - 1) + (a_4 - 1) + (a_6 - 1) \ldots (a_{98} - 1) +$
$a_2 + a_4 + a_6 + \ldots a_{98}$

Let $X = a_2 + a_4 + a_6 + \ldots a_{98}$

$137 = 2X - 49$

$186 = 2X$

$93 = X$

Note w/ geo

$a_1 + a_1 r^2 + a_1 r^4$

$a_1 r + a_1 r^3 + a_1 r^5$

$\hookrightarrow r(a_1 + a_1 r^2 + a_1 r^4)$

4) Let a1 be the first term of the geo sequence. Let r be the common ratio.

$7 = a_1 + a_1 r + a_1 r^2 + \ldots$

$-3 = \quad a_1 r \quad + a_1 r^3 + a_1 r^5 + \ldots$

Let $X = a_1 + a_1 r^2 + a_1 r^4 + \ldots$

$= 4(7 - 3)$

$$3 = r(a_1 + a_1 r^2 + a_1 r^4 + \dots)$$

$$3 = r4$$

$$r = 3/4$$

$$7 = \frac{a_1}{1-r} \Rightarrow 7 = \frac{a_1}{1-\frac{3}{4}}$$

$$7 = \frac{a_1}{\frac{1}{4}}$$

$$7 = 4a_1$$

$$a_1 = 7/4.$$

$$a_1 + r = \frac{7}{4} + \frac{3}{4} = \frac{5}{2}$$