

Arithmetic/Geometric Series

Monday, September 21, 2020 2:25 PM

4, 7, 10, 13, 16, ...

Arithmetic sequence is a series of numbers where the difference between successive terms is the same.

We typically denote a_1 as the first term in the sequence and d as the common difference.

$$a_1 = 4, d = 3$$

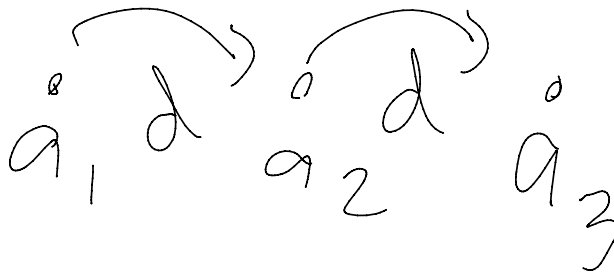
Types of things we want to be able to do with them:

- 1) Given a term in the sequence and the common difference, find another term in the sequence
- 2) Find the sum of some set of terms in the sequence. (Find the sum of the first n terms of the sequence where n is given to you.)
- 3) If you know two terms in the sequence, what is the common difference?

1) Example: $a_{15} = 18, d = 2$, what is a_{100} ?

$$a_2 = a_1 + d$$

$$a_3 = a_2 + d = a_1 + 2d$$



#hops
 a_n to a_m
is $m - n$.

$$a_{100} = a_{15} + (100-15)d = 18 + 85 \times 2 = 188$$

Example: $a_1 = 5, d = 3$, what is a_{100} ?

$$a_{100} = a_1 + (100-1)d = 5 + 99 \times 3 = 302$$

2) Sum of an arithmetic sequence: $a_1, a_2, a_3, \dots, a_n$.

How to figure out $a_1 + a_2 + a_3 + \dots + a_n$.

$\frac{n}{2}$

2) Sum of an arithmetic sequence. $a_1, a_2, a_3, \dots, a_n$.

How to figure out $a_1 + a_2 + a_3 + \dots + a_n$.

$$S = 4 + 7 + 10 + \dots + 100 \text{ (33 terms)}$$

$$S = 100 + 97 + 94 + \dots + 4$$

$$2S = 104 + 104 + 104 + \dots + 104$$

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$$2S = 104 \times 33, \text{ so } S = (104 \times 33)/2$$

More generally first term is a_1 , last term is a_n and there are n columns

$$2S = (a_1 + a_n)n$$

$$S = \frac{(a_1 + a_n)n}{2}$$

$$a_n = a_1 + (n-1)d$$

$$S = \frac{(2a_1 + (n-1)d)n}{2}$$

3) $a_{10} = 50, a_{40} = -10$

$$a_{40} = a_{10} + (40 - 10)d$$

$$-10 = 50 + 30d$$

$$-60 = 30d$$

$$d = -2$$

Geometric sequences -

3, 6, 12, 24, 48, ...

There is a common ratio between successive terms.

For this sequence $a_1 = 3, r = 2$

- 1) Given a term in the sequence and the common ratio, find another term in the sequence
- 2) Finding the sum of the first n terms of the sequence
- 3) Finding the sum of an infinite geometric sequences

4) Given two terms, determine the common ratio.

In general, the terms of a geometric sequence are:

$$a_1, a_1r, a_1r^2, a_1r^3, \dots$$

The n th term is $a_n = a_1r^{n-1}$.

$$a_3 = 384, r = 1/2, \text{ what is } a_8?$$

$$a_8 = a_3r^5 = 384(1/2)^5 = 384/32 = 12$$

Finding the sum of the first n terms of a geometric sequence

$$\begin{aligned} S &= a_1 + a_1/r + a_1/r^2 + \dots + a_1/r^{n-1} \\ -rS &= a_1/r + a_1/r^2 + \dots + a_1/r^n \end{aligned}$$

$$S - rS = a_1 - a_1/r^n$$

$$S(1-r) = a_1(1 - r^n)$$

$$S = \frac{a_1(1-r^n)}{1-r}, \quad r \neq 1$$

$a_1 = 384, r = 1/2$, what is the sum of the first 10 terms

$$= \frac{384(1 - \frac{1}{2}^{10})}{1 - \frac{1}{2}}$$

$$= \frac{384 \times \frac{1023}{1024}}{1}$$

$$= \frac{3 \frac{1}{2}}{2 \times 384 \times 1023}$$

$$= \frac{3069}{4}$$

Consider an infinite geometric sequence where $|r| < 1$, then as n goes to infinite the term ar^{n-1} goes to 0, limit wise, so all the same work I did above is valid, just ignore that last term:

$$S = \frac{a_1 \text{ (gone)}}{1-r}$$

What is $3 + 2 + 4/3 + 8/9$, etc. $a_1 = 3, r = 2/3$

$$S = \frac{3}{1 - \frac{2}{3}} = 9$$

4) $a_4 = 6$ and $a_{10} = 384$, what is the common ratio?

$$a_{10} = a_4 r^{10-4}$$

$$384 = 6r^6$$

$$64 = r^6$$

$$r = 2 \text{ or } -2$$