Exam 1 Review

Sunday, September 13, 2020 7:09 AM

Exam will be short answer style questions, similar to what you've seen on homework and quizzes.

Exam will be in 3 parts:

1. Logic
2. Sets
3. Recitation (Roots of Equations, D = RT, Logs)

Exam will be given as short timed assignments in the Exam Section.

Date of Exam: Thursday, September 17, 2020
Exam Aids: Printed Class Notes, Formula Sheet, Calculator (but you must show every step, no skipping steps with the Calc)

Note: **All times are Eastern Standard Time and PM**

Part 1 will be released 1:30, **Due 1:55**, with Late Due 2:05 (pretend this is 2)
Part 2 will be released 1:55**, Due 2:20**, with Late Due 2:30 (pretend this is 2:25)
Part 3 will be released 2:20**, Due 2:50**, with Late Due 3:00 (pretend this is 2:55)

File types accepted: .doc, .docx, .pdf and .txt

My recommendation is to directly type your answers either in the doc I post or in a text file, or your own doc. **I do NOT recommend writing and scanning.**

Stuff People Have Messed Up with my Online Exams
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1. **Worked till too late and missed the deadline.**
2. **Turned in the wrong document.**
3. **Scanner not working, Stylus not readable.**

Sample Exam 1 from Spring 2020
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1) Truth Tables - just plug in the definitions of and, or, xor, implication, not and

BE REALLY CAREFUL!!!

2) This is a cool question, an analysis question…my strategy…look at 13 and first try to disprove since that's faster…if you find a counter example and get lucky great! If not, move onto the next one and try to find a counter example there.

   a. Our strategy to finding a counter-example was making the if false, and then then true. This is the only case where the implication is one way. So, we designed our counter example so that the then is true, we did this by setting q = True, r = False. (I want it to be "barely" true so that I can get the if to be false.) I see that the first clause is automatically true, so I focus on the second clause. I see that if I set p = True, I make the whole second clause false, which makes the whole if False, Bingo! Got my counter example: p = True, q = True, r = False.

   b. We want to prove #14 via the laws of logic only to prove the two way equivalence.

$p \rightarrow (q \wedge r)$

$\overline{p} \vee (q \wedge r)$ Def Imp

$(\overline{p} \vee q) \wedge (\overline{p} \vee r)$ Dist

$(p \rightarrow q) \wedge (p \rightarrow r)$ Def Imp

3) for all x show there exist a y. Universe = R^+

$$\frac{x}{y} + \frac{y}{x} = 4$$

$$\frac{x^2 + y^2}{xy} = 4$$

$\wedge y$

$$x^2 + y^2 = 4xy$$

$$\frac{x^2 - 4xy + y^2 = 0}{y^2}$$

$$\left(\frac{x}{y}\right)^2 - 4\left(\frac{x}{y}\right) + 1 = 0$$

$$(x-(2+\sqrt{3})y)(x+(2-\sqrt{3})y) = 0$$

Let $z = \frac{x}{y}$

$$z^2 - 4z + 1 = 0$$

$$z = \frac{4 \pm \sqrt{16-4}}{2}$$

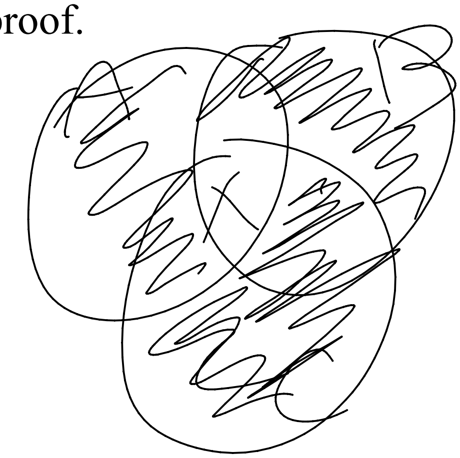
$$= \boxed{2 \pm \sqrt{3}}$$

$$\frac{x}{y} = 2 \pm \sqrt{3}$$

$$\boxed{y = \frac{x}{2 \pm \sqrt{3}}}$$

$$y = \frac{x}{2 \pm \sqrt{3}}$$

4) Use a venn diagram to get a feel for if it's true or not, but don't use the venn diagram in your proof.



My proof - Set Table, then use words to explain why you don't want a row for A = 1, B = 1, C = 1. (This is disallowed by the if, since we only care about situations where that intersection is empty.) Then, if the two columns in question are equal, the two sets are equal under the given restriction in the if.

First we show that LHS is a subset of the RHS
Let x be an arbitrary element of A union B union C.

Case 1: Let x belong to A or B (or maybe both A and B) but not C

Case 2: Let x belong to A or C (or maybe both A and C) but not B

Case 3: Let x belong to B or C (or maybe both B and C) but not A

Case 1: If x is not in C and x is in B, then it's in B - C.
Alternatively, of x isn't in B, then it must be in A to be in this case and is consequently in A - B.

Case 2: If x is not in B and x is in A, then it's in A - B.

Alternatively, if x isn't in A it must be in C to be in this case and consequently is in C - A.

Case 3: If x is not in A and x is in C, then it's in C - A.
Alternatively, fi x isn't in C it must be in B to be in this case and consequently is in B - C.

Now we show an arbitrarily chosen element of the RHS belongs to the LHS.

Let x be an arbitrarily chosen element of A-B union B-C union C-A. We must show it belongs to A union B union C.

Case 1: x is in A - B, so x is in A and not in B, by def on union it must be in A union B union C.

Case 2: x is in B - C, so x is in B and not in C, by def on union it must be in A union B union C.

Case 1: x is in C - A, so x is in C and not in A, by def on union it must be in A union B union C.

5) This is something true with numbers, so you should be very suspicious. It's not true with sets. Lets make a counter-example.

   B-A = D-C (want to make true)
   B-D = A-C (want to make this false)

   Key observation: using the empty set is really useful in counter-examples!!!

   B = {}, D = {} (This automatically guarantees the truth of the statement we want to make true.)
   B - D = {}
   We just need A - C to not be empty:
   A = {1}, C = {}

   This statement is false. Consider the following counter example:

Let A = {1}, B = C = D = {}.

B - A = {}, D - C = {}, so the if is true.

But

B - D = {} and A - C = {1}, so the then is false.

6) Key here, figure out what to assign variables to, figure out what quantity to express in two ways.

Let r = avg speed biking

We will find two ways to equate the total time of the triathlon.

Total distance = 9
Total avg speed = 4.5
Total time = 9 miles/4.5 miles per hour = 2 hours

Time run + Time bike + Time swim = 2 hours

3 miles/8 mph + 3 miles/r + 3 miles/2 mph = 2 hours
15/8 hours + 3 miles/r = 2 hours
3 miles/r = 1/8 hour
r = 24 miles/hour

8) Multiply a4 by the same number three times to get a7 (def of a geo sequence. Let the common ratio be r.

48*(r^3) = 384
r^3 = 8
r = 2

Sequence is 6,12,24,48,96,192,384, etc.
Divide each term by 13 and see if there is a pattern for the remainder.

6, 12, 11, 9, 5, 10, 7, 1, 2, 4, 8, 3, 6, repeats…

We want term number 2020.
Repeats every 12.
12 | 2020
168 R 4

This means that $b_{2016} = 3$, $b_{2017} = 6$, $b_{2018} = 12$, $b_{2019} = 11$ and $b_{2020} = 9$

Note: Remainder when 24 x 2 is divided by 13 is the same as the remainder when 11 x 2 is divided by 13. We can always substitute any value in the expression that is not in an exponent with an equivalent mod and still get the same remainder…

24 x 2 = (13 + 11) x 2 = 13 x 2 + 11 x 2, yellow part drops out of remainder calculation.