$b^c = a$ the equivalent log statement is

$\log_b a = c$.

$2^6 = 64$

$\log_2 64 = 6$

$\log_2 64$ is asking "What power do you raise 2 to, to obtain 64?"

$$2^{log_2 64} = 64$$

Is this a coincidence, or is

$$a^{log_a b} = b$$

Always true?

The latter is always true because it's self-referential. A to some power equals ? The power to which we raise a to, is exactly the power that when you raise a to that power, you obtain b.

Just the same $\log_a a^b = b$.

log a + log b = log ab

This comes from the exponential formula

$c^a c^b = c^{a+b}$

First multiply c by itself a times, then multiply c by itself b times. In total, we've multiplied c by itself a+b times, which is the right hand side.

Let $x = \log_c a$, and let $y = \log_c b$

Thus, $c^x = a$ and $c^y = b$

Now consider $ab = c^x c^y = c^{x+y}$

Now, rewrite this as a log statement:

$\log_c ab = x + y$

Now, substitute for x and y:

$\log_c ab = \log_c a + \log_c b$

Apply this rule over and over again with the same item:

$\text{Log}_c a + \log_c a + \log_c a \ldots + \log_c a = \log_c a^n = n\log_c a$.

LHS has n terms the same, RHS is using log addition rule

So this is the derivation of the power rule.

log a – log b = log (a/b), can be proven pretty similarly to addition proof.

Log change of base

$\log_a b = \frac{log_c b}{log_c a}$

For example, $\log_8 x = \frac{log_2 x}{log_2 8} = \frac{log_2 x}{3}$.

What is $\log_8 16$? $8^x = 16$, $2^{3x} = 2^4$, so 3x = 4 and x = 4/3

1) What is the value of $(81^{log_3 1234})^{0.25}$?

$$(3^{4log_3 1234})^{0.25}$$

$$3^{4log_3 1234(0.25)}$$

$$3^{log_3 1234} = 1234$$

Recall that $(a^b)^c = a^{bc}$, this says multiply a by itself b times, then take that number and multiply it by itself c times: $a^b a^b a^b \ldots a^b$ (c times). How many times is a multiplied by itself? Bc.

2)Determine the ordered pair, (a, b), that satisfies the following pair of equations:

$$log_{16} a^2 + log_8 b^3 = 11$$

$$log_8 a^6 + log_{16} b^{10} = 32$$

Note: You may express a and b as a some base raised to an exponent instead of a single value, if you wish.

$$2log_{16} a + 3log_8 b = 11$$
$$\frac{2log_2 a}{log_2 16} + \frac{3log_2 b}{log_2 8} = 11$$

$$\frac{2log_2 a}{4} + \frac{3log_2 b}{3} = 11$$

$$6log_8 a + 10log_{16} 6 = 32$$

$$\frac{6log_2 a}{3} + \frac{10log_2 b}{4} = 32$$

$$\frac{2log_2 a}{4} + \frac{3log_2 b}{3} = 11$$

Let X = $log_2 a$, Y = $log_2 b$

$$2x + \frac{5}{2}y = 32$$

$$2x + 4y = 44$$

Subtract second from the first: $-\frac{3}{2}y = -12$, y = 8, x= 6

Finally a = 2^6 = 64, b = 2^8 = 256

3) What is the value of a for which $\sum_{i=2}^{10} \frac{1}{log_i a} = 1$?

$$\frac{1}{log_2 a} + \frac{1}{log_3 a} + \frac{1}{log_4 a} + \cdots + \frac{1}{log_{10} a} = 1$$

$$\frac{1}{\frac{lna}{\ln 2}} + \frac{1}{\frac{lna}{\ln 3}} + \frac{1}{\frac{lna}{\ln 4}} + \cdots + \frac{1}{\frac{lna}{\ln 10}} = 1$$

$$\frac{ln2}{\ln a} + \frac{ln3}{\ln a} + \frac{ln4}{\ln a} + \cdots + \frac{ln10}{\ln a} = 1$$

$$log_a 2 + log_a 3 + log_a 4 + \cdots + log_a 10 = 1$$

$$log_a 10! = 1$$

So a = 10!

4) The sequence $\log_{12} 162$, $\log_{12} x$, $\log_{12} y$, $\log_{12} z$, , $\log_{12} 1250$ is an arithmetic progression. What is x?

Let $D = \log_{12} x - \log_{12} 162$, the common difference of the arithmetic progression.

$\text{Log}_{12}162 + 4D = \log_{12}1250$

$4D = \log_{12}1250 - \log_{12}162$

$4D = \log_{12}(1250/162) = \log_{12}(625/81) = \log_{12}(5^4/3^4)$

$4D = \log_{12}(5/3)^4$

$4D = 4\log_{12}(5/3)$

$D = \log_{12}(5/3)$

$\text{Log}_{12}162 + \log_{12}(5/3) = \log_{12}x$

$\text{Log}_{12}270 = \log_{12}x$, so $x = 270$.