COT 3100 Recitation #3: Logs - Solutions for Recitation Problems

Problems For Recitation

1) Solve for x in the following equation: $log_2 x^3 + log_4(8x^2) = -\frac{5}{2}$

Solution

$$log_2 x^3 + log_4(8x^2) = -\frac{5}{2}$$

$$3log_2 x + log_4 8 + log_4 x^2 = -\frac{5}{2}$$
$$3log_2 x + \frac{3}{2} + 2log_4 x = -\frac{5}{2}$$
$$3log_2 x + 2(\frac{log_2 x}{log_2 4}) = -4$$
$$3log_2 x + 2(\frac{log_2 x}{2}) = -4$$

$$4log_2 x = -4$$

$$log_2 x = -1$$

So, $x = 2^{-1} = \mathbf{\frac{1}{2}}$.

Note: We can evaluate $log_4 8$, by solving $4^x = 8$, so $2^{2x} = 2^3$, since the exponential function is one to one, we know that $2x = 3$, so $x = \frac{3}{2}$.

2) If $3^{log_9 x} = 9^{log_3 27}$, what is the prime factorization of x?

Solution

$$3^{log_9 x} = 9^{log_3 27}$$
$$3^{log_9 x} = 9^3$$
$$3^{log_9 x} = (3^2)^3$$
$$3^{log_9 x} = 3^6$$

Since the exponential function is one to one, we know that:

$$log_9 x = 6$$

It follows that $x = 9^6 = (3^2)^6 = \mathbf{3^{12}}$, in prime factorized form.

3) What is the value of the following summation?

$$\sum_{i=1}^{n} log_{n!} i$$

Solution

$$\sum_{i=1}^{n} log_{n!} i = log_{n!}\left(\prod_{i=1}^{n} i\right) = log_{n!} n! = \mathbf{1}$$

Note: The sigma means to add the terms log 1 + log 2 + log 3 + … + log n, where the base is n!. The log addition rule says that to put these in a single log, we multiply each item we are taking the log of. The pi sign means exactly that, to multiply each term you get when you plug i into the expression on the inside and the product of each integer from 1 to n is defined as n factorial (n!).

4) Define a function f as follows: f(1) = 2. For all integers n > 1, $f(n) = (f(n-1))^{2^n}$.

What is the value of $log_{65536}(log_{65536}(f(4))$?

Express your answer in the form $2^a + 2^b$, where both a and b are integers. Note that $65536 = 2^{16}$.

Solution

First, let's calculate f(4). $f(2) = f(1)^4 = 2^4$, $f(3) = (f(2))^8 = (2^4)^8 = 2^{32}$, $f(4) = f(3)^{16} = (2^{32})^{16} = 2^{512}$.

Let X = $log_{65536}(f(4))$. Then, using the power rule, we have X = $512 log_{65536} 2$. Use the log base change rule with base 2 to get the following:

$$log_{65536} 2 = \frac{log_2 2}{log_2 65536} = \frac{1}{16}$$

Thus, X = $\frac{512}{16} = 32$.

The value we desire is = $log_{65536} X = log_{65536} 32 = \frac{log_2 32}{log_2 65536} = \frac{5}{16} = \mathbf{2^{-2} + 2^{-4}}$.