

COT 3100 Recitation #2: D = RT Quiz and Solutions
9/4-8/2020

1) Express $\log_{16}32$ as a fraction in lowest terms. What is the sum of the numerator and denominator of this fraction?

Solution

By definition of log, we seek the value of a variable x such that $16^x = 32$. Rewriting this equation using a base of 2 for both sides we get:

$$(2^4)^x = 2^5$$

$$2^{4x} = 2^5$$

$$4x = 5$$

$$x = \frac{5}{4}$$

2) Solve for x in the following equation: $\log_5(25x^3) + \log_{25}(5x^2) = 14.5$

Solution

Using log rules we get:

$$\log_5(25x^3) + \log_{25}(5x^2) = 14.5$$

$$\log_5 25 + \log_5 x^3 + \log_{25} 5 + \log_{25} x^2 = 14.5$$

$$2 + 3\log_5 x + 1/2 + 2\log_{25} x = 14.5$$

$$3\log_5 x + 2\log_{25} x = 12$$

If we convert the second log to base 5, we get:

$$3\log_5 x + \frac{2\log_5 x}{\log_5 25} = 12$$

$$3\log_5 x + \frac{2\log_5 x}{2} = 12$$

$$4\log_5 x = 12$$

$$\log_5 x = 3$$

So, $x = 5^3 = \underline{\underline{125}}$.

3) What is the value of the following expression:

$$(\log_2 3) \times (\log_3 4) \times (\log_4 5) \times (\log_5 6) \times (\log_6 7) \times (\log_7 8)?$$

Solution

Use a common base for each log expression (here we'll use the base of e) to get the equivalent expression:

$$\frac{\ln 3}{\ln 2} \times \frac{\ln 4}{\ln 3} \times \frac{\ln 5}{\ln 4} \times \frac{\ln 6}{\ln 5} \times \frac{\ln 7}{\ln 6} \times \frac{\ln 8}{\ln 7} = \frac{\ln 8}{\ln 2} = \log_2 8 = 3$$

4) Define a function f as follows: $f(1) = 2$. For all integers $n > 1$, $f(n) = f(n-1)^{f(n-1)}$. What is the value of $\log_2(\log_2(f(4)))$?

Solution

$$f(2) = 2^2 = 4$$

$$f(3) = 4^4 = (2^2)^4 = 2^8 = 256$$

$$f(4) = (2^8)^{256}$$

We must take the log base 2 if this value twice, let's do it once:

$$\log_2(2^8)^{256} = \log_2 2^{8 \times 256} = 8 \times 256 = 2^3 \times 2^8 = 2^{11}$$

Now, take log base 2 of this value to obtain the final answer:

$$\log_2 2^{11} = \underline{\underline{11}}$$

5) Find the value of x which satisfies the following equation, where a is an arbitrary positive value:

$$\log_2 a = \log_4 a + \log_8 a + \log_x a$$

Solution

Subtract the first two terms on the right to the left:

$$\log_2 a - \log_4 a - \log_8 a = \log_x a$$

Get a common base of 2:

$$\log_2 a - \frac{\log_2 a}{\log_2 4} - \frac{\log_2 a}{\log_2 8} = \log_x a$$

$$\log_2 a - \frac{\log_2 a}{2} - \frac{\log_2 a}{3} = \log_x a$$

$$\log_2 a \left(1 - \frac{1}{2} - \frac{1}{3}\right) = \log_x a$$

$$\log_2 a \left(\frac{6-3-2}{6} \right) = \log_x a$$

$$\log_2 a \left(\frac{1}{6} \right) = \log_x a$$

$$\log_2 a = 6 \log_x a$$

Use a common base for both logs, say base e:

$$\frac{\ln a}{\ln 2} = 6 \left(\frac{\ln a}{\ln x} \right)$$

$$\ln x = 6 \ln 2$$

$$\ln x = \ln 2^6$$

Thus, $x = 2^6 = \underline{\mathbf{64}}$.