COT 3100 Recitation #2: D = RT - Solutions for Recitation Problems

1) Casey drove a total of 100 miles. For the first portion of the trip she averaged 40 miles per hour and for the second/last portion of the trip she averaged 55 miles per hour. If her average speed for the entire trip was 50 miles an hour, how long (in miles) was the first portion of her trip? Please express your answer as a fraction in lowest terms.

Solution

Casey drove a total of 100 miles. For the first portion of the trip she averaged 40 miles per hour and for the second/last portion of the trip she averaged 55 miles per hour. If her average speed for the entire trip was 50 miles an hour, how long (in miles) was the first portion of her trip? Please express your answer as a fraction in lowest terms.

Although the distance of the first portion of the trip is unknown, lots of other stuff is known. It's immediately clear that we should let d = the distance of the first portion of the trip and that the second portion of the trip has distance 100 - d. Now, we can let t1 = the time for the first portion of the trip and t2 = time for the second portion of the trip and t3 = time for the full trip. Since we know the average speed and distance of the whole trip, we get the following equations for the two portions of the trip and the whole trip itself:

d = 40(t1)
100 - d = 55(t2)
100 = 50 (t3)

Also, by definition, we have:

t3 = t2 + t1

Using the last equation, we get t3 = 2. We can also solve for t2 and t1 as follows:

$$t1 = \frac{d}{40}$$

$$t2 = \frac{100-d}{55}$$

Now, just plug into that last equation to get:

$$2 = \frac{d}{40} + \frac{100-d}{55}$$

From here, it's just algebra for one equation with one variable. Get a common denominator first:

$$2 = \frac{11d}{40(11)} + \frac{8(100-d)}{(8)55}$$

Now, multiply by that denominator:

$$2(40)(11) = 11d + 800 - 8d$$

$(80)(11) = 3d + 80(10)$
$3d = 80(11) - 80(10)$
$3d = 80(11 - 10)$
$3d = 80$
$d = \mathbf{\frac{80}{3}}$

So, the key here was identifying three trips (portion1, portion2, whole) and the associated variables. Then, creating each of the relevant equations and attempting to reduce the number of variables as fast as possible. This left us with one equation in one variable, which is ideal. Also, notice some of the algebra above - multiplications were not done. Instead expressions were factored in the hope that stuff would drop out, which it did! This is key for exams w/o calculators.

2) Jessica is going from Orlando to Miami and she takes a 15 minute break at Vero Beach. Her goal is to average 60 miles per hour for the whole trip. The distance between Orlando and Vero Beach is 100 miles and the distance between Vero Beach and Miami is 140 miles. If her average driving speed from Orlando to Vero Beach is 50 miles per hour, how fast must her average speed be driving from Vero Beach to Miami to achieve her goal?

Solution

The total distance to drive is 240 miles and if we aim to achieve an average of 60 miles per hour, we must make the drive in 240 miles/60 mph = 4 hours.

If we drive 100 miles averaging 50 miles per hour, we take 100 miles/50 mph = 2 hours.

Then we take a break for 15 minutes. This means we've used 2 hours and 15 minutes.

We have 4 hours - (2 hours 15 minutes) = 1 hour 45 minutes = $\frac{7}{4}$ hours to complete the trip.

Let r be the average speed we drive from Vero Beach to Orlando. Then we have:

$$140\ miles = r(\frac{7}{4}hours)$$
$$r = 140 \times \frac{4}{7} mph = \mathbf{80\ mph}$$

3) The current in a river is flowing steadily at 3 miles per hour. A motor boat which travels at a constant rate in still water goes downstream 4 miles and then returns to its starting point. The trip takes one hour, excluding the time spent in turning the board around. What is the ratio of the downstream to the upstream rate?

Solution

Let the rate of the motor boat in still water be r miles per hour. In total the motor boat travels 8 miles, traveling 4 of those miles at (r+3) mph and traveling the other 4 miles at (r - 3) mph. The question asks us to find the ration (r+3) to (r-3). Our d = rt equation when equating time is:

$$1 = \frac{4}{r+3} + \frac{4}{r-3}$$

$$1 = \frac{4(r-3)}{(r+3)(r-3)} + \frac{4(r+3)}{(r+3)(r-3)}$$

$$(r+3)(r-3) = 4r - 12 + 4r + 12$$

$$r^2 - 9 = 8r$$

$$r^2 - 8r - 9 = 0$$

$$(r-9)(r+1) = 0$$

$r = 9$, since r is positive.

It follows that the desired ratio is (9+3):(9-3) or **2: 1.**

4) Selena and Trina live 13 miles apart. Yesterday Selena started to ride her bicycle toward Trina's house. A little later Trina started to ride her bicycle towards Selena's house. When they met, Selena had ridden for twice the length of time as Trina and at four-fifths of Mike's rate. How many miles had Trina ridden when they met?

Solution

Let Trina's rate be *r* and the amount of time Trina rides the bicycle be *t*. Selena's rate is $\frac{4}{5}r$ and the amount of time she rode her bicycle is *2t*. Summing the distances they rode we get:

$$13 = rt + \frac{4}{5}r(2t)$$

$$13 = \frac{13}{5}rt$$

$$rt = 5$$

Since the distance Trina rode is *rt*, the desired answer is **5 miles.**