**COT 3100 Recitation #1: Polynomial Review Quiz and Solutions**
**8/28-9/1/2020**

1) What is the sum of the roots of the equation $3x^2 - 15x + 7 = 0$?

**Solution**
The sum of the roots of the quadratic $ax^2 + bx + c = 0$ is $-b/a$. Thus, for this equation the sum of the roots is $\boldsymbol{-\left(-\frac{15}{3}\right) = 5}$.

2) What is the product of the roots of the equation $x^2 + 8x + 10 = 0$?

**Solution**
The product of the roots of the quadratic $ax^2 + bx + c = 0$ is $c/a$. Thus, for this equation the sum of the roots is $\boldsymbol{\frac{10}{1} = 10}$.

3) What is the following product: $(2x^4 + x - 7)(3x^3 + 4x + 6)$?

**Solution**
$(2x^4 + x - 7)(3x^3 + 4x + 6) =$ $(6x^7 + 8x^5 + 12x^4) +$
$(3x^4 + 4x^2 + 6x) +$
$(-21x^3 - 28x - 42)$

$= \mathbf{6x^7 + 8x^5 + 15x^4 - 21x^3 + 4x^2 - 22x - 42}$

4) Let r and s be the roots of the equation $x^2 - 4x + 2 = 0$. What is the quadratic equation with leading coefficient 1 with roots $r^2$ and $s^2$?

**Solution**
If an equation has roots $r^2$ and $s^2$, then the sum of those roots is $r^2 + s^2$ and the product of those roots is $r^2s^2$. If we can find the values of these two expressions, we can recreate the desired quadratic.

Using the given information, we see that:

$r + s = 4$
$rs = 2$

So, take the first equation and square it and simplify:

$(4)^2 = (r + s)^2 = r^2 + 2rs + s^2 = r^2 + 2(2) + s^2$

Now, solve for $r^2 + s^2$:

$16 = r^2 + 4 + s^2$
$12 = r^2 + s^2$

Since rs = 2, it follows that $(rs)^2 = r^2s^2 = (2)^2 = 4$.

Thus, the desired quadratic is $\mathbf{x^2 - 12x + 4 = 0}$.

5) If $x + \frac{1}{x} = 10$, what is $x^3 + \frac{1}{x^3}$?

**Solution**
Cube the given expression:

$$(x + \frac{1}{x})^3 = 10^3$$

$$x^3 + 3x + \frac{3}{x} + \frac{1}{x^3} = 1000$$

$$x^3 + 3(x + \frac{1}{x}) + \frac{1}{x^3} = 1000$$

But we know what $x + \frac{1}{x}$ is, so let's substitute for that:

$$x^3 + 3(10) + \frac{1}{x^3} = 1000$$

$$x^3 + \frac{1}{x^3} = \mathbf{970}$$