COT 3100 Recitation #1: Polynomial Review - Solutions for Recitation Problems

1) Compute the sum of all of the roots in the equation $2x^2 - 5x + 7 = 0$.

Solution

The sum of the roots of the quadratic $ax^2 + bx + c = 0$ is $-b/a$. Thus, for this equation the sum of the roots is $-\left(-\frac{5}{2}\right) = \frac{5}{2}$.

2) Compute the following product $(2x^3 - 7x^2 + 3x - 1)(4x^2 + 2x - 5)$

Solution

$(2x^3 - 7x^2 + 3x - 1)(4x^2 + 2x - 5) =$

$(8x^5 + 4x^4 - 10x^3) +$
$(-28x^4 - 14x^3 + 35x^2) +$
$(12x^3 + 6x^2 - 15x) +$
$(-4x^2 - 2x + 5)$

$=$ $\mathbf{8x^5 - 24x^4 - 12x^3 + 37x^2 - 17x + 5}$

3) The r and s be the roots of the quadratic equation $x^2 + 7x - 11 = 0$. What is the quadratic equation with leading coefficient 1 that has roots r^2 and s^2?

Solution

If an equation has roots r^2 and s^2, then the sum of those roots is $r^2 + s^2$ and the product of those roots is r^2s^2. If we can find the values of these two expressions, we can recreate the desired quadratic.

Using the given information, we see that:

$r + s = -7$
$rs = -11$

So, take the first equation and square it and simplify:

$(-7)^2 = (r + s)^2 = r^2 + 2rs + s^2 = r^2 + 2(-11) + s^2$

Now, solve for $r^2 + s^2$:

$49 = r^2 - 22 + s^2$
$71 = r^2 + s^2$

Since $rs = -11$, it follows that $(rs)^2 = r^2s^2 = (-11)^2 = 121$.

Thus, the desired quadratic is $\mathbf{x^2 - 71x + 121 = 0}$.

4) If $x + \frac{1}{x} = 7$, what is $x^3 + \frac{1}{x^3}$?

Solution

Cube the given expression:

$$(x + \frac{1}{x})^3 = 7^3$$

$$x^3 + 3x + \frac{3}{x} + \frac{1}{x^3} = 343$$

$$x^3 + 3(x + \frac{1}{x}) + \frac{1}{x^3} = 343$$

But we know what $x + \frac{1}{x}$ is, so let's substitute for that:

$$x^3 + 3(7) + \frac{1}{x^3} = 343$$

$$x^3 + \frac{1}{x^3} = \mathbf{322}$$