

COT 3100 Fall 2020 Homework #10
Please Consult WebCourses for the due date/time

1) (14 pts) Let R_1 and R_2 be relations on a set $A = \{1, 2, 3, 4\}$.

In particular, let $R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 4), (3, 4), (4, 2), (4, 3), (4, 4)\}$ and
 $R_2 = \{(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (4, 1), (4, 2)\}$.

Determine the following:

- a) Whether or not R_1 is reflexive, irreflexive, symmetric, anti-symmetric and transitive or not.
- b) Whether or not R_2 is reflexive, irreflexive, symmetric, anti-symmetric and transitive or not.
- c) The relation $R_1 \circ R_2$.
- d) The relation $R_2 \circ R_1$.
- e) $R_1 \cup R_2$
- f) $R_1 \cap R_2$
- g) The reflexive, symmetric and transitive closures of both R_1 and R_2 .

2) (5 pts) Let R be a relation over the positive integers defined as follows:

$$R = \{(a,b) \mid 2b < a < 3b\}$$

Determine whether or not R satisfies the following properties. Give a brief justification for each of your answers.

- (i) reflexive
- (ii) irreflexive
- (iii) symmetric
- (iv) anti-symmetric
- (v) transitive

3) (6 pts) How many symmetric relations on the set $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ contain the ordered pairs $(2, 3)$, $(3, 2)$, $(4, 7)$, $(5, 5)$ and $(8, 7)$?

4) (8 pts) Let the relation R , over the set of 8-bit unsigned integers (0 to 255), be defined as

$$R = \{(a,b) \mid a \wedge b \equiv 0 \pmod{8}\}$$

Note: The carat symbol represents a bitwise XOR. Prove that R is an equivalence relation. How many equivalence classes does R have? What would be an easier way of stating this exact same relation that doesn't use the bitwise XOR operator?

5) (4 pts) Let $f(x) = x^2 + 4x - 45$ with a domain of all real $x \in [-\infty, -2]$. Prove that f is injective. What is the range of f ? (You may either use Calculus or complete the square to prove your answers.)

6) (4 pts) Find $f^{-1}(x)$ for the function given in question #5.

7) (4 pts) Let $f(x) = \sqrt{e^x}$ and $g(x) = x^2$. Determine $h_1(x) = f(g(x))$ and $h_2(x) = g(f(x))$. What are the largest possible domains for which functions $h_1(x)$ and $h_2(x)$ can be defined?

8) (5 pts) Please give a summary of the life and mathematical contributions of Emmy Noether. Please aim for a length of roughly 200 - 400 words. **Your summary must be typed.** Please state the sources you used in writing your summary.