COT 3100 Fall 2020 Homework #7
Please Consult WebCourses for the due date/time

1) (9 pts) After catching sight of a particularly adorable online video, the Small Monster collector has begun a second career as a Small Monster researcher, intending to develop a taxonomic argument that marmosets are not in fact conventional animals, but are in fact Small Monsters. As a warm-up to their argument, they are reminding themselves of the biological family to which marmosets belong.

a. Taking into account identical letters, how many ways can the researcher arrange the word CALLITRICHIDAE?
b. Taking into account identical letters, how many ways can the researcher arrange the word CALLITRICHIDAE that start or end with vowels?
c. Taking into account identical letters, how many ways can the researcher arrange the word CALLITRICHIDAE that contain the substring LITERAL?

2) (2 pts) While they were researching marmosets, a particularly convoluted walk through Wikipedia caused the researcher to develop a fascination with the politics of religion in England in the 18^{th} century. Taking into account identical letters, how many ways are there to arrange the letters in the word ANTIDISESTABLISHMENTARIANISM?

3) (15 pts) A Small Monster collector is visiting the Knightrola region, bringing 30 Small Monster Containment Devices with them. The devices are highly reliable and can effectively be assumed to always work. **How many different types of Small Monsters can the collector capture under the following conditions?**

a. The collector has access to Grass, Space, Electric Scooter, Mask, Citrus, and Rubber Duck type Small Monsters
b. The above, and intends to capture *at least* one of each of those types
c. Both of the above, and intends to capture *at least* five Space types
d. All of the above, and intends to capture *at most* four Electric Scooter types
e. All of the above, and intends to capture *at most* three Rubber Duck types

4) (4 pts) A tournament has n teams, ranked 1 to n. In the first match, the n^{th} ranked team plays the $(n-1)^{th}$ ranked team. The loser of that match finishes in last place. The winner of that match plays the $(n-2)^{th}$ ranked team. The loser of that match finishes in second to last place. The winner of that match goes on to play the $(n-3)^{th}$ ranked team, and so forth. The last match will pit the previous match winner against the first ranked team. The winner of this last match wins the tournament and the loser of this last match places second in the tournament. In how many different orders can the n teams finish. (Note: count two orders as different if at least one team receives a different finishing position in the two orders.) As a quick example, in a tournament with three teams, A, B and C, with A ranked first, B ranked second and C ranked third, here are the following four possible finishing orders: (A, B, C), (A, C, B), (B, A, C) and (C, A, B). Note that A can't finish worse than second place, since A only plays in the last match and is guaranteed to be first or second.

5) (4 pts) How many integers in the range 1 to 100,000 inclusive are divisible by 6, 15 or 21?

6) (4 pts) A school's Student Government Association (SGA) has four specific positions: President, Vice President, Treasurer and Secretary. In addition, the, SGA has 12 members chosen at large. All of the seats must be filled by students from the SGA class, of which there are 50. No student may hold more than 1 position. How many different ways are there to fill the positions? (Note: Two ways are considered different if there is a person in one way who has a ***different role*** or no role in the other way. For example, if Jamila is President in one way and she is Vice President in another, those two ways must count differently. But if she is an at large member in both ways, that is not proof that those ways should be counted differently.)

7) (7 pts) While the Small Monster collector, researcher, and ecclesiastical historian was busy with marmosets, we showed in class that we can an assign an integer order to the members of an infinite two-dimensional array; and thus, that infinite two-dimensional arrays are countable, and can be re-ordered as infinite one-dimensional arrays. Using that result, prove by induction that for any positive integer n, an infinite n-dimensional array is countable.

8) (5 pts) Give a summary of the academic contributions of Georg Cantor. Please aim for a length of roughly 200 - 400 words. ***Your summary must be typed.*** Please state the sources you used in writing your summary.