

COT 3100 Fall 2020 Homework #6
Please Consult WebCourses for the due date/time

1) Prove by induction on n that, for all positive integers n :

$$\sum_{i=1}^n (i(i!)) = (n+1)! - 1$$

2) Prove by induction on n that, for all positive integers n :

$$\sum_{i=1}^n i6^i = \frac{6(5n6^n - 6^n + 1)}{25}$$

3) Prove by induction on n that, for all positive integers n , $21|(4^{n+1} + 5^{2n-1})$.

4) Let t_n be a sequence defined as follows: $t_0 = 5$, $t_1 = 12$, and for all $n \geq 2$, $t_n = 5t_{n-1} - 6t_{n-2}$. Using strong induction on n with 2 base cases, prove that for all non-negative integers n ,

$$t_n = 3(2^n) + 2(3^n).$$

5) Using induction on n , prove for all positive integers n that $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^n = \begin{bmatrix} F_{2n+1} & F_{2n} \\ F_{2n} & F_{2n-1} \end{bmatrix}$, where F_n denotes the n^{th} Fibonacci number. (Recall that the Fibonacci numbers are defined as follows: $F_0 = 0$, $F_1 = 1$, and for all $n > 1$, $F_n = F_{n-1} + F_{n-2}$.)

6) Give a summary of the academic contributions of Grigori Perelman. Please aim for a length of roughly 200 - 400 words. **Your summary must be typed.** Please state the sources you used in writing your summary.