

COT 3100 Fall 2020 Homework #5
Please Consult WebCourses for the due date/time

- 1) Determine the following summation in terms of n : $\sum_{i=1}^{n^2} (2i + 5)$.
- 2) Determine the following infinite summation: $\sum_{i=1}^{\infty} (3i) \left(\frac{2}{3}\right)^i$.
- 3) Let $g(n)$ be defined as follows be a function defined on the positive integers as follows:

$$g(1) = 3, g(2) = 2, g(3) = 4$$
$$\text{For all } n > 3, g(n) = g(n-1) + 2g(n-2) + 3g(n-3).$$

What are the values of $g(4)$, $g(5)$ and $g(6)$?

If you would like for fun, write a computer program which prints out the first 1000 values of $g(n) \bmod 10^9 + 7$. Feel free to include the source code inside the document containing your homework solutions.

- 4) Determine the following matrix product, $\begin{bmatrix} 2n + 1 & 7 \\ -n + 1 & 3 \end{bmatrix} \begin{bmatrix} 6 & n + 1 \\ 3n - 1 & 2 \end{bmatrix}$, in terms of n .
- 5) Let n be a positive integer such that $7 \mid (8^n - 1)$. Prove that $7 \mid (8^{n+1} - 1)$. (Hint: Rewrite $8^{n+1} - 1$, where a portion of the expression is 8 times $8^n - 1$.)
- 6) Recall that the Fibonacci numbers are defined as follows:

$$F_0 = 0, F_1 = 1, \text{ for all integers } n > 1, F_n = F_{n-1} + F_{n-2}.$$

Using induction on n , prove that $\sum_{i=0}^n F_i = F_{n+2} - 1$.

- 7) Give a summary of the academic contributions of Dr. Ingrid Daubechies. Be sure to include information about wavelets in your write up. Please aim for a length of roughly 200 - 400 words. **Your summary must be typed.** Please state the sources you used in writing your summary.