COT 3100 Fall 2020 Homework #5
Please Consult WebCourses for the due date/time

1) Determine the following summation in terms of n: $\sum_{i=1}^{n^2}(2i+5)$.

Solution

$$\sum_{i=1}^{n^2}(2i+5) =$$
$$\sum_{i=1}^{n^2}2i + \sum_{i=1}^{n^2}5 =$$
$$2*\sum_{i=1}^{n^2}i + \sum_{i=1}^{n^2}5 =$$
$$2*\frac{(n^2+1)n^2}{2} + 5*n^2 =$$
$$n^4 + n^2 + 5n^2 =$$
$$\underline{\boldsymbol{n^4 + 6n^2}}$$

2) Determine the following infinite summation: $\sum_{i=1}^{\infty}(3i)(\frac{2}{3})^i$.

Solution

$$\sum_{i=1}^{\infty}(3i)(\frac{2}{3})^i =$$
$$3*\sum_{i=1}^{\infty}(i)(\frac{2}{3})^i$$

Let $r = \frac{2}{3}$

Let $S_{ag} = \sum_{i=1}^{\infty}(i)(r)^i$

$$S_{ag} = 1*r^1 + 2*r^2 + 3*r^3 + 4*r^4 + \cdots$$
$$r*S_{ag} = 1*r^2 + 2*r^3 + 3*r^4 + 4*r^5 + \cdots$$
$$S_{ag} - r*S_{ag} = 1*r^1 + 1*r^2 + 1*r^3 + 1*r^4 + \cdots$$

$1*r^1 + 1*r^2 + 1*r^3 + 1*r^4 + \cdots$ is a geometric sum which equates to $\frac{r}{1-r}$

$$(1-r)S_{ag} = \frac{r}{1-r}$$
$$S_{ag} = \frac{r}{(1-r)^2}$$
$$3*\sum_{i=1}^{\infty}(i)\left(\frac{2}{3}\right)^i = 3*\frac{\frac{2}{3}}{\left(1-\frac{2}{3}\right)^2} = 3*\frac{\frac{2}{3}}{\frac{1}{9}} = \mathbf{18}$$

3) Let g(n) be defined as follows be a function defined on the positive integers as follows:

g(1) = 3, g(2) = 2, g(3) = 4
For all n > 3, g(n) = g(n-1) + 2g(n-2) + 3g(n-3).

What are the values of g(4), g(5) and g(6)?

Solution

g(1) = 3
g(2) = 2
g(3) = 4
g(4) = g(3) + 2g(2) + 3g(1) = 4 + 2*2 + 3*3 = 4 + 4 + 9 = **17**
g(5) = g(4) + 2g(3) + 3g(2) = 17 + 2*4 + 3*2 = 17 + 8 + 6 = **31**
g(6) = g(5) + 2g(4) + 3g(3) = 31 + 2*17 + 3*4 = 31 + 34 + 12 = **77**

If you would like for fun, write a computer program which prints out the first 1000 values of g(n) mod 10^9+7. Feel free to include the source code inside the document containing your homework solutions.

4) Determine the following matrix product, $\begin{bmatrix} 2n+1 & 7 \\ -n+1 & 3 \end{bmatrix}\begin{bmatrix} 6 & n+1 \\ 3n-1 & 2 \end{bmatrix}$, in terms of n.

Solution

$$\begin{bmatrix} 2n+1 & 7 \\ -n+1 & 3 \end{bmatrix}\begin{bmatrix} 6 & n+1 \\ 3n-1 & 2 \end{bmatrix} =$$
$$\begin{bmatrix} (2n+1)(6)+(7)(3n-1) & (2n+1)(n+1)+(7)(2) \\ (-n+1)(6)+(3)(3n-1) & (-n+1)(n+1)+(3)(2) \end{bmatrix} =$$
$$\begin{bmatrix} 12n+6+21n-7 & 2n^2+2n+n+1+14 \\ -6n+6+9n-3 & -n^2-n+n+1+6 \end{bmatrix} =$$
$$\begin{bmatrix} \mathbf{33n-1} & \mathbf{2n^2+3n+15} \\ \mathbf{3n+3} & \mathbf{-n^2+7} \end{bmatrix}$$

5) Let n be a positive integer such that 7 | (8^n-1). Prove that 7 | (8^{n+1} - 1). (Hint: Rewrite 8^{n+1} -1, where a portion of the expression is 8 times 8^n - 1.)

Solution

$$8^{n+1}-1 =$$
$$8*8^n-1 =$$
$$8*8^n-(8-7) =$$
$$8*8^n-8+7 =$$
$$8(8^n-1)+7 =$$

Using premise: 7 | (8^n-1) → $(8^n-1) = 7c$, where c is an integer

$$8(7c)+7 =$$
$$7(8c+1)$$
$$7|7(8c+1)$$
$$8^{n+1}-1 = 7(8c+1)$$
$$7|(8^{n+1}-1)$$

6) Recall that the Fibonacci numbers are defined as follows:

$F_0 = 0$, $F_1 = 1$, for all integers n > 1, $F_n = F_{n-1} + F_{n-2}$.

Using induction on n, prove that $\sum_{i=0}^{n} F_i = F_{n+2} - 1$.

Solution

Base case: n = 0

LHS = $\sum_{i=0}^{0} F_i = F_0 = 0$

RHS = $F_2 - 1 = 1 - 1 = 0$

LHS = RHS

Inductive Hypothesis: Assume for an arbitrarily chosen non-negative integer n=k that

$$\sum_{i=0}^{k} F_i = F_{k+2} - 1$$

Inductive Step: Prove that $\sum_{i=0}^{k+1} F_i = F_{k+3} - 1$

$$\sum_{i=0}^{k+1} F_i =$$

$$\sum_{i=0}^{k} F_i + F_{k+1} =$$

Using IH: $\sum_{i=0}^{k} F_i = F_{k+2} - 1$

$$F_{k+2} - 1 + F_{k+1} =$$

Using definition of Fibonacci numbers: $F_{k+1} + F_{k+2} = F_{k+3}$

$$\boldsymbol{F_{k+3} - 1}$$

The Base case and the Inductive Step are proven, so by induction, $\sum_{i=0}^{n} F_i = F_{n+2} - 1$ holds true for all integers $n \geq 0$

7) Give a summary of the academic contributions of Dr. Ingrid Daubechies. Be sure to include information about wavelets in your write up. Please aim for a length of roughly 200 - 400 words. ***Your summary must be typed.*** Please state the sources you used in writing your summary.

Sample Summary

Dr. Daubechies is currently the James B. Duke professor for Duke University's Electrical and Computer Engineering program. She had a fascination with numbers form an extremely young age in her native Belgium, where she did her schooling, culminating in earning her Ph. D. in theoretical physics in 1980 from the Free University in Brussels.

She started off her career with a professorship at Vrije Universiteit Brussel in the early 1980s, moving to a researcher position at the Courant Institute of Mathematical Sciences in 1986, where she discovered that wavelets could be utilized to aid digital signal processing. It's from this work that she developed what is now known as the Daubechies wavelet and the biorthogonal CDF wavelet. The JPEG 2000 standard uses a wavelet from this group, and this is what she's best known for. In addition to wavelets, she has used mathematics and technology to automatically extract information from bones and teeth and she's created advanced image processing techniques to verify the authenticity of famous paintings.

Over the course of her academic career, Daubechies has taught at Rutgers, Princeton and now Duke. She has routinely helped other women in mathematics and science. She is on the board of the Enhancing Diversity in Graduate Education (EDGE) program that helps women applying to graduate school and she started a summer program at Duke in mathematics for girls who are rising high school seniors. She has won many awards over her illustrious career, including the North American Laureate in 2019. The award has been given annually since 1988 to five women in the fields of chemistry, physics, materials science, mathematics and computer science.

Source: https://en.wikipedia.org/wiki/Ingrid_Daubechies