Fall 2020 COT 3100 Homework #3

1) (5 pts) State the quotient and remainder when dividing a by b for each of the following examples:

1) a = 147, b = 6
2) a = 12345, b = 106
3) a= 77, b = 96
d) a = 421, b = 77
e) a = 176, b = 148

2) (6 pts) Using the method of calculating remainders for one full cycle of exponents, find the remainders when dividing a by b for each of the following examples:

1) $a = 7^{15}$, b = 5
2) $a = 11^{21}$, b = 7
3) $a = 13^{30}$, b = 11

Note: What simplification can you make for each base before even building the answers for one full cycle?

3) (6 pts) Using the method of fast modular exponentiation, find the remainders when dividing a by b for each of the following examples:

1) $a = 2^{22}$, b = 13
2) $a = 3^{31}$, b = 17

4) (16 pts) Use the Euclidean Algorithm, showing every step, to calculate each of the following greatest common divisors (gcd).

1) gcd(123, 67)
2) gcd(609, 377)
3) gcd(198, 135)
4) gcd(7238, 923)

5) (5 pts) Determine, with proof, all ordered pairs of integers (x, y) which satisfy the equation

$$93411x + 2844y = 12345.$$

Hint: To save time, use common divisibility rules instead of the Euclidean Algorithm.

6) (7 pts) Let x and y be integers such that $17 \mid (2x + 4y)$. Prove that $17 \mid (28x + 5y)$.

7) (5 pts) Give a summary of the life and mathematical contributions of Andrew Wiles.