# Fall 2020 COT 3100 Homework #1

1) (5 pts) Multiply $(x^4 + 5x^3 + 2x) \times (3x^2 - 4x + 7)$. What is the fully expanded result?

2) (4 pts) Create a truth table for the following logical expression: $(p \wedge q) \vee \neg r$. Please make sure your first three columns are for p, q and r, respectively, and that you have appropriate intermediary columns. In addition, make sure that the rows are ordered in numerical order, in binary, if we treat False as the bit 0 and True as the bit 1.

3) (8 pts) Create a truth table for the following logical expression: $\neg(p \vee q) \wedge (r \oplus s)$. Please make sure your first three columns are for p, q and r, respectively, and that you have appropriate intermediary columns. In addition, make sure that the rows are ordered in numerical order, in binary, if we treat False as the bit 0 and True as the bit 1.

4) (6 pts) If it's raining, the ground outside is wet. However, if the ground outside is wet, that does not necessarily mean it's raining. So, we can say that the fact it's raining *implies* that the ground outside is wet – but not vice versa. **Give at least three** real-life examples of similar situations where, in formal terms, $p \rightarrow q$ but $\neg(q \rightarrow p)$: where a condition $p$ necessarily implies a condition $q$, but the reverse is not true.

- For each example, **explain** why $q$ must be true if $p$ is true.
- For each example, **give a situation** where $q$ can be true without $p$ being true.

5) (6 pts) Use the truth table method to prove the following two expressions are logically equivalent:

$$(p \vee q) \rightarrow r$$

$$(p \rightarrow r) \wedge (q \rightarrow r)$$

6) (6 pts) Use the laws of logic to show that two following expressions are logically equivalent:

$$(p \wedge (\bar{p} \rightarrow r)) \vee ((q \wedge s) \vee (q \wedge \bar{s}))$$

$$p \vee q$$

7) (5 pts) Use the rules of inference to prove the following argument:

$p \vee q$
$p \rightarrow r$
$q \rightarrow r$
$s \vee t$
$\bar{t}$
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$\therefore r \wedge s$

8) (5 pts) Prove or disprove the following claim over the universe of all real numbers for x and y.

$$\forall x\left[\exists y[x^2 - 5xy + 6y^2 = 0]\right]$$

If the claim is false, find a value of x for which it is false. If it is true, show which value(s) of exist to make the claim true.

9) (5 pts) The last question of each homework assignment will be to write up a two paragraph summary of a topic from the history of mathematics. The idea here is that rarely is any of this history taught in mathematics classes and while I don't have class time to teach it, I thought it would be nice if students learned a bit for each homework assignment. There's no need to use fancy sources, websites will do, but please site which websites you pulled your information from.

Give a summary of the life and mathematical contributions of Sophie Germain. If you are so inclined, for fun, write a program that prints out all Sophie Germain primes less than 1,000,000. Include your source code in your .pdf submission. (Please submit C, Java or Python code.) Note, no points are assigned for the program.