

COT 3100 Section 2 Final Exam - Part A Solution

1) (10 pts) A course grade is comprised of 4 Quizzes, 5 Homework Assignments and 3 Exams. The quizzes are worth 20% of the course grade, the homework assignments are worth 30% of the course grade and the exams are worth the remaining 50% of the course grade. Each of the quizzes is out of 30 points, each of the homework assignments is out of 50 points and each of the exams is out of 100 points. Each quiz is worth the same amount of the course grade, each homework assignment is worth the same amount of the course grade and each exam is worth the same amount of the course grade. Note that a quiz, homework assignment and exam are each worth different amounts of the course grade. Here are Selenia's grades on the assignments that have already been turned in:

Quiz 1: 22/30	Homework 1: 45/50	Exam 1: 60/100
Quiz 2: 15/30	Homework 2: 30/50	
Quiz 3: 0/30	Homework 3: 42/50	

As discussed many times in class, Webcourses incorrectly calculates a course grade in the situation when it's known how many assignments in each category there will be for the whole course. We discussed how to properly calculate the course grade in these instances. Thus, as promised, do the following:

- Calculate Selenia's ACTUAL current course average, which only counts the current assignments, properly weighted.
- Selenia's grade as it would be calculated on Webcourses.

In order to get any credit on this question, you must show your work and your answers must be perfectly accurate, since incorrect methods will naturally lead to answers that are "pretty close."

Solution

(a) Note that each quiz is worth 5% of the course grade, each homework assignment is worth 6% of the course grade and the exams are worth (50/3)% of the course grade. It follows that each quiz point is (1/6)% of the course grade, each homework point is (6/50)% of the course grade and each exam point is (1/6)% of the course grade. The actual course grade, with $15\% + 18\% + 50/3\% = (149/3)\%$ of the total course grade is calculated as follows:

$$\frac{(22 + 15 + 0) \left(\frac{1}{6}\right) + (45 + 30 + 42) \left(\frac{6}{50}\right) + 60 \left(\frac{1}{6}\right)}{\frac{149}{300}} = \frac{300 \left(\frac{37}{6} + \frac{351}{25} + 10\right)}{149} = \frac{9062}{149}$$

Written as a regular percentage, rounded to two decimal spots, we get **60.82%**

(b) Webcourses calculates the percentage in each category and weights those categories based on the final weighting percentages. So the quiz percentage is $\frac{37}{90} \times 100 = \frac{370}{9}$, the total homework percentage is $\frac{117}{150} \times 100 = 78\%$ and the total exam percentage is 60%. Thus, Webcourses would do the following calculation:

$$.2 \times \frac{370}{9} + .3 \times 78 + .5 \times 60 = \frac{74}{9} + \frac{234}{10} + 30 = \frac{740 + 2106 + 2700}{90} = \frac{2773}{45}$$

Written as a regular percentage, rounded to two decimal spots, we get **61.62%**

Grading: 5 pts for both averages, give partial credit as you see fit. There are lots of equivalent algebraic ways to do what I show above, so be careful to give credit to alternative ways that work.

2) (10 pts) Let r and s be the roots of the quadratic equation $x^2 - 13x + 9 = 0$. Find the quadratic equation with the roots $2r+s$ and $r + 2s$ with a leading coefficient of 1. (Note: Full credit will only be given for the technique where the value of neither r nor s is ascertained.)

Solution

Given that the roots of the first equation are r and s , using the formula for the sum and product of the roots, we have that

$$r + s = 13$$

$$rs = 9$$

Now, we want to find the sum and product of the roots of a quadratic whose roots are $2r+s$ and $r+2s$:

$$(2r + s) + (r + 2s) = 3r + 3s = 3(r+s) = 3(13) = 39$$

$$(2r + s)(r + 2s) = 2r^2 + 5rs + 2s^2 = 2r^2 + 4rs + 2s^2 + rs = 2(r^2 + 2rs + s^2) + rs = 2(r + s)^2 + rs = 2(13)^2 + 9 = 347.$$

It follows that the desired quadratic equation is $x^2 - 39x + 347 = 0$.

Grading: 2 pts for writing down what $r+s$ is and what rs is.

1 pt for stating that we need to know $(2r+s) + (r+2s)$

1 pt for stating that we need to know $(2r+s) * (r+2s)$

1 pt for finding $(2r+s) + (r+2s)$

4 pts for finding $(2r+s) * (r+2s)$

1 pt for giving the final quadratic

3) (10 pts) For this question, assume that if x_1 ounces of liquid 1 at temperature t_1 is mixed with x_2 ounces of liquid 2 at temperature t_2 , the resulting mixture is $x_1 + x_2$ ounces with a temperature of $\frac{t_1x_1+t_2x_2}{x_1+x_2}$. Arup's wife, Anita, likes her tea mixed with milk. (She also likes it with sugar, but I didn't want to make this problem too hard!) Unfortunately, she has a very specific preference for both the ratio of tea to milk and the temperature of the overall mixture. Specifically, she wants a 6:1 ratio of tea to milk at a temperature of 180° F. On Arup's first attempt, he mixed 8 ounces of tea at 200° F with half an ounce of milk at 40° F. It's not possible for Arup to add any more tea to the mixture, but he is allowed to add more milk and he can heat that milk to any temperature greater than 40° F, if necessary, before he adds it. How much milk should Arup add to the tea and at what temperature should he heat it before adding it, to fix the tea?

Solution

Since the mixture must have 8 ounces of tea and the ratio of tea to milk is 6:1, it follows that the total amount of milk to add to the mixture is $8/6 = 4/3$ ounces. Currently $1/2$ an ounce was added, so the total amount of milk that must be added is $4/3 - 1/2 = 5/6$ ounces.

The current temperature of the mixture is $\frac{200(8)+40(.5)}{8.5} = \frac{1620}{8.5} = \frac{3240}{17}$ degrees Fahrenheit.

Let t be the temperature of the $5/6$ added ounces of milk. We know we want this temperature to also equal 180 degrees Fahrenheit. Set these two expressions equal to one another:

$$\frac{\frac{3240}{17} \times \frac{17}{2} + t \times \frac{5}{6}}{\frac{28}{3}} = 180$$

$$\frac{3(1620 + t \times \frac{5}{6})}{28} = 180$$

$$4860 + \frac{5t}{2} = 5040$$

$$\frac{5t}{2} = 180$$

$$t = 180 \times \frac{2}{5} = 72$$

It follows that Arup should add 5/6th of an ounce of milk to the tea, heating it to a temperature of 72° F, before adding it.

Grading: 3 pts for figuring out that 5/6 ounces of milk needs to be added.

3 pts for figuring out that the current temperature is $\frac{3240}{17}$.

4 pts for figuring out the final answer.

Note: You can skip part 2 by solving this $\frac{200 \times 8 + 40 \times \frac{1}{2} + t \times \frac{5}{6}}{\frac{19}{2}} = 180$. If they do this, then 7 pts for solving this equation.