COP 3502 Section 2 Exam #1 - Part 3 (Recitation Material) - 25 pts Solution

Date: 9/17/2020

Start Time: 2:20 pm EST
End Time: 2:45 pm EST

1) (8 pts) Let r and s be the roots of the equation $x^2 - 8x + 3 = 0$. What is the quadratic equation with leading coefficient of 1 with the roots $\frac{r}{s}$ and $\frac{s}{r}$?

Solution

The desired answer is of the form $x^2 - ax + b = 0$, where $a = \frac{r}{s} + \frac{s}{r}$, and $b = \frac{r}{s} \times \frac{s}{r}$, based on the sum and product of the roots form of a quadratic. Let's solve for both:

$$a = \frac{r}{s} + \frac{s}{r} = \frac{r^2+s^2}{rs} = \frac{(r+s)^2-2rs}{rs} = \frac{8^2-6}{3} = \frac{58}{3}$$

$$b = \frac{r}{s} \times \frac{s}{r} = 1$$

It follows that the desired quadratic is $\boldsymbol{x^2 - \frac{58}{3}x + 1 = 0}$. An quadratic with the same roots but integer coefficients is $3x^2 - 58x + 3 = 0$.

Grading: 1 pt for identifying which two expressions needed to be evaluated to get the appropriate quadratic, 2 pts for calculating b, 5 pts for calculating a (1 pt for denominator, 4 pts for numerator)

2) (8 pts) Dave starts running in a straight line at a constant rate of 20 feet per second. Two minutes later, Ayesha starts from the same spot Dave started at, attempting to catch up with him, running at 25 feet per second. How many **minutes** after Ayesha starts running will she catch up with Dave?

Solution

In 2 minutes, which is 120 seconds, Dave, running at 20 feet/second, will run 120x20 = 2400 feet.

Since Ayesha runs 5 feet per second faster than Dave, she'll reduce her distance from Dave at a rate of 5 feet per second. She must reduce this gap of 2400 feet to 0 feet. Let t be the time in seconds it takes for her to reduce the gap to 0. Thus we get the equation:

2400 feet = (5 feet per second)t
t = 2400/5 seconds = 480 seconds

Converted to minutes, this is **8 minutes** (divide by 60 since there are 60 seconds in a minute to obtain this.)

Grading: 3 pts to get distance of Dave's head start, 3 pts to calculate # of seconds Ayesha takes to catch up, 2 pts to convert back to minutes.

3) (8 pts) Find the ordered pair (a, b) which satisfies the following pair of equations?

$$2log_9 a^3 + 3log_{27} b^2 = 18$$

$$5log_{27} a^2 + 4log_9 b^3 = \frac{94}{3}$$

<u>Solution</u>

First, convert all of the logs to base 3 and apply the log exponent rule:

$$\frac{6log_3 a}{log_3 9} + \frac{6log_3 b}{log_3 27} = 18$$

$$\frac{10log_3 a}{log_3 27} + \frac{12log_3 b}{log_3 9} = \frac{94}{3}$$

Now, simplify the logs:

$$\frac{6log_3 a}{2} + \frac{6log_3 b}{3} = 18$$

$$\frac{10log_3 a}{3} + \frac{12log_3 b}{2} = \frac{94}{3}$$

Let X = $log_3 a$ and Y = $log_3 b$, and simplify the constants to get:

$$3X + 2Y = 18$$
$$10X + 18Y = 94$$

Multiply the first equation through by -9:

$$-27X - 18Y = -162$$
$$10X + 18Y = 94$$

Add the two equations to get:

$$-17X = -68$$
$$X = 4$$

Substitute in the equation in yellow to solve for Y: 3(4) + 2Y = 18, 2Y = 6, Y = 3.

Now, we can solve for a and b:

4 = $log_3 a$, so a = 3^4 = 81 and 3 = $log_3 b$, so b = 3^3 = 27. The desired ordered pair is **<u>(81, 27).</u>**

Grading: 4 pts to convert to base 3, 4 pts to solve the system of equations (2 pts subtracting stuff, 2 pts converting from 4 and 3 to 81 and 27, respectively.) Note: I put in lots of extra steps and commentary that students don't need to put in. Give credit if it's clear they know what they are doing but maybe calculated a couple things in their head, like $log_3 27$.

4) (1 pt) A unicorn company is a privately held start up company valued at $1 billion or more. How many horns does the mythical creature with the same name have?

One   **Grading: Give to all.**