# <u>COP 3502 Section 2 Exam #1 - Part 2 (Sets) - 25 pts Solution</u>

**Date: 9/17/2020**

**Start Time: 1:55 pm EST**
**End Time: 2:20 pm EST**

1) (8 pts) Prove or disprove the following assertion for all sets A, B and C:

$$if\ A \subseteq \bar{C}, then\ B - (A - C) \subseteq B - (C - A)$$

**<u>Solution</u>**
This assertion is false.

Consider the following counter-example:

Let A = {1}, B = {1,2}, and C = {2}.

In this example $A \subseteq \bar{C},$ holds since all of A's elements do NOT belong to C and thus do belong to the complement of C.

A - C = {1}, since A contains 1 and C does not.
C - A = {2}, since C contains 2 and A does not.
B - (A - C) = {2}, since B contains 2 and A-C does not.
B - (C - A) = {1}, since B contains 1 and C-A does not.

Thus, in this counter example, the set B - (A-C) is NOT a subset of B - (C - A), since the former set contains 2 while the latter set does not.

**Grading: 3 pts for saying it's false (If they say it's true max of 2 pts use you judgement),**
**3 pts for clearly stating a counter-example (give only 2 of these pts if invalid)**
**2 pts for showing that the counter-example makes the if true and the then false.**
**These points can only be earned if the counter example is valid.**

2) (8 pts) Prove or disprove the following assertion for all sets A and B:

$$if\, \wp(A) \subseteq \wp(B), then\ A \subseteq B$$

**Solution**

We will prove the contrapositive of this statement to prove the original statement. Namely, we will show the following:

$$if\ A \nsubseteq B, then\ \wp(A) \nsubseteq \wp(B)$$

We will prove this statement via direct proof. Thus, we assume that $A \nsubseteq B$.
By definition of subset, this means there exists some element x such that $x \in A \wedge x \notin B$.
Given this information, it follows that the set $\{x\} \in \wp(A)$, while $\{x\} \notin \wp(B)$.
Since we've found an element that belongs to $\wp(A)$, but doesn't belong to $\wp(B)$, we may conclude that $\wp(A) \nsubseteq \wp(B)$.

**Grading: 2 pts for describing how they will prove. 1 pt for first step regardless of the proof type (for mine it's assuming A isn't a subset of B). 1 pt for the next step (above that's stating there's an x in A that isn't in B. 2 pts for the following observation that {x} is in P(A) but not in P(B), must use set notation to get these two points (key idea here is that power sets contain sets not elements). 2 pts for the conclusion.**

3) (9 pts) Let A, B and C be sets that satisfy the following equations dealing with set cardinalities:

$$|A \cap B \cap C| = 4$$
$$|A| + |B| + |C| = 59$$
$$|B \cup C| = 33$$

Using this information, determine $|A \cup (B \cap C)|$.

**Solution**

$|A \cup (B \cap C)| = |A| + |B \cap C| - |A \cap (B \cap C)|$ , applying Inclusion-Exclusion Principle

$= |A| + |B| + |C| - |B \cup C| - |A \cap (B \cap C)|$, applying I/E Principle on the second set.

$= 59 - 33 - 4 = 22$ , plugging in give information.

**Grading: 3 pts for first step I/E**
**4 pts for applying I/E to B intersect C**
**2 pts for plugging in numbers and simplifying to the answer**