# COP 3502 Section 2 Exam #1 - Part 1 (Logic) - 25 pts Solutions

**Date: 9/17/2020**

**Start Time: 1:30 pm EST**
**End Time: 1:55 pm EST**

1) (8 pts) Complete the following truth table. Please write T for true and F for false. It's preferred that you type into the grid given. If you make your own grid, please keep the order of the rows the same and make each entry legible. (Note: There is a not symbol above the xor'ed term.)

**Solution**

| $p$ | $q$ | $r$ | $\overline{p \oplus r}$ | $q \wedge \bar{r}$ | $(\overline{p \oplus r}) \vee (q \wedge \bar{r})$ |
|---|---|---|---|---|---|
| F | F | F | **T** | **F** | **T** |
| F | F | T | **F** | **F** | **F** |
| F | T | F | **T** | **T** | **T** |
| F | T | T | **F** | **F** | **F** |
| T | F | F | **F** | **F** | **F** |
| T | F | T | **T** | **F** | **T** |
| T | T | F | **F** | **T** | **T** |
| T | T | T | **T** | **F** | **T** |

**Grading: 1 pt per row, row must be completely correct to earn the point.**

2) (7 pts) Using the laws of logic, show that the following logical expression is a tautology:

$$p \vee (q \rightarrow r) \vee (\bar{p} \wedge (q \vee \bar{p}))$$

Please use the format shown in class to express your answer.

**Solution**

| | |
|---|---|
| $p \vee (q \rightarrow r) \vee (\bar{p} \wedge (q \vee \bar{p})) \leftrightarrow$ | |
| $p \vee (\bar{q} \vee r) \vee (\bar{p} \wedge (q \vee \bar{p})) \leftrightarrow$ | **Implication Identity** |
| $p \vee (\bar{q} \vee r) \vee (\bar{p} \wedge (\bar{p} \vee q)) \leftrightarrow$ | **Commutative Law** |
| $p \vee (\bar{q} \vee r) \vee (\bar{p}) \leftrightarrow$ | **Absorption Law** |
| $(p \vee (\bar{p})) \vee (\bar{q} \vee r) \leftrightarrow$ | **Commutative Law** |
| $T \vee (\bar{q} \vee r) \leftrightarrow$ | **Inverse Law** |
| $(\bar{q} \vee r) \vee T \leftrightarrow$ | **Commutative Law** |
| $T$ | **Domination Law** |

**Grading: If roughly correct, take off 1 pt per incorrect step, note that none of the commutative steps are necessary, it's okay if students skip those. Take off 2 pts total if all reasons are missing, take off 1 pt total if some reasons are missing or incorrect.**

**If mostly not correct, give a max of 3 pts, 1 pt for each correctly applied rule.**

3) (5 pts) Prove or disprove the following assertion over the universe of integers:

$$\exists x \forall y [xy = 0]$$

Please clearly note whether the assertion is true or not, followed a justification of your answer. Most of the points are awarded for the justification.

**<u>Solution</u>**
**This is true. The x that exists is 0. If we let x = 0, then for all y, xy = 0y = 0, as desired.**

**Grading: 1 pt for stating it's true, 3 pts for stating that the x that exists is 0, and 1 pt for showing that when x is 0, xy = 0 as well, for all y.**

4) (5 pts) Prove or disprove the following assertion over the universe of real numbers:

$$\forall x \exists y [xy = 1]$$

Please clearly note whether the assertion is true or not, followed a justification of your answer. Most of the points are awarded for the justification.

**<u>Solution</u>**
**This is false. To prove it's false we must find a counter-example. Namely, we must find one value of x for which the statement is not true. That value of x is x = 0.**

**If we set x = 0, then there does not exist a value of y such that xy = 1, because 0y = 0, and can not equal 1, no matter what we set y equal to.**

**Grading: 1 pt for stating it's false, 3 pts for stating that the x that exists to prove it's not true is 0, and 1 pt for showing that when x is 0, xy can not equal 1.**