Fall 2019 COT 3100 Homework 8 Solutions

1) **In a tennis league, there are eight teams of 7 players each. A set of 10 players is selected randomly out of the 56 players in the league. What is the probability that at least one player from each team will be selected?**

Each team has 7 player's and you we want to pick at least one player of each team, so let's say we choose one player from each team, we are going to end up with 8 players since there are 8 teams, but we want 10 players so let's say we pick 3 players from a particular team instead of 1. So we have 1 player from 7 teams and 3 players the last team. The number of ways of doing this can be represented mathematically as

$$\binom{7}{1}\binom{7}{1}\binom{7}{1}\binom{7}{1}\binom{7}{1}\binom{7}{1}\binom{7}{1}\binom{7}{3}$$

Now this is only for when we pick 3 out of the last team, but we could pick 3 players from any team, so to get the amount of times to pick 3 players out of one team and 1 from the others you would use the equation

$$\binom{8}{1}\binom{7}{1}^7\binom{7}{3}$$

Since there are 8 options, 7 repetitions of $\binom{7}{1}$ and 1 repetition of $\binom{7}{3}$, and you can permute that $\frac{8!}{7!1!}$ ways. Now we should note that you can also choose 1 player from 6 teams and 2 players from 2 other teams and this would be expressed like this

$$\binom{8}{2}\binom{7}{1}^6\binom{7}{2}^2$$

Since we pick 1 player from 6 teams and 2 from 2 teams and the $\binom{8}{2}$ is there, because there are 8 options 6 repetitions of $\binom{7}{1}$ and 2 repetitions of $\binom{7}{2}$ which would mean $\frac{8!}{6!2!}$ combinations. Now there is no other way of picking 10 players where there is at least 1 from each team, if we want to pick 2 from one team we need to pick 2 from another team in order to get 10 players and at least one from each team, but no more, if we pick 3 from 1 team, we need to pick 1 from all other teams, if we wanted to pick 4 , we would need to not choose a person from one team, and that is not admissible under the condition that we need to have at least 1 player from every team.

Therefore, the total number of combinations for picking 10 players with at least one player from each team is

$$\binom{8}{1}\binom{7}{1}^7\binom{7}{3}+\binom{8}{2}\binom{7}{1}^6\binom{7}{2}^2$$

Now to get the probability of picking 10 players with at least one player from each team we need to divide the result by the sample space, and the sample space is

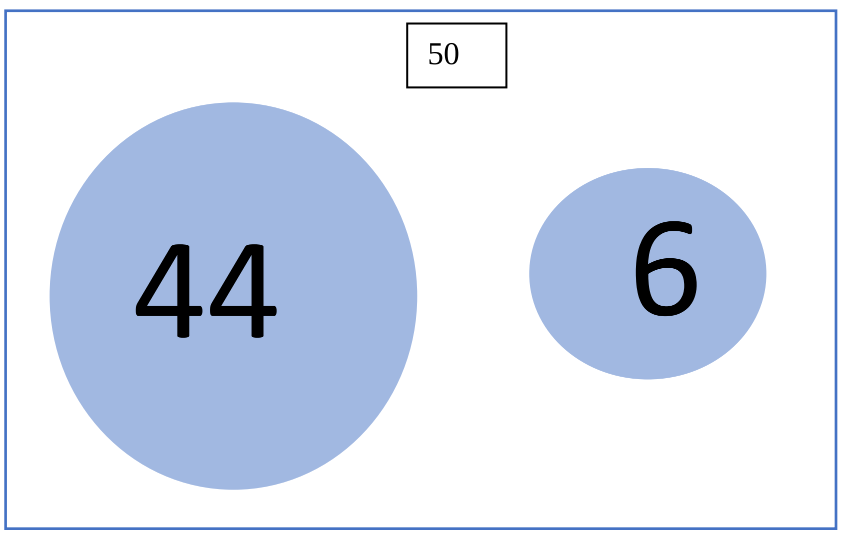
$$\binom{56}{10}$$

So, the probability of picking 10 players with at least 1 player from each team is

$$\frac{\binom{8}{1}\binom{7}{1}^7\binom{7}{3} + \binom{8}{2}\binom{7}{1}^6\binom{7}{2}^2}{\binom{56}{10}}$$

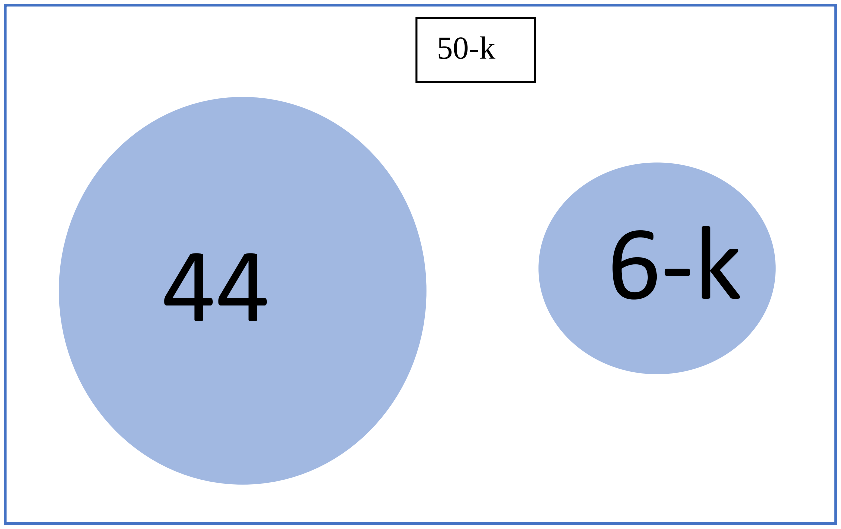
2) **50 tickets are sold in a raffle where 6 prizes will be given. Terri buys 10 of the tickets. What is the probability that Terri wins k prizes, where k is an integer in between 0 and 6, inclusive? Please give your answer in terms of k.**

To understand this problem better let's draw a diagram

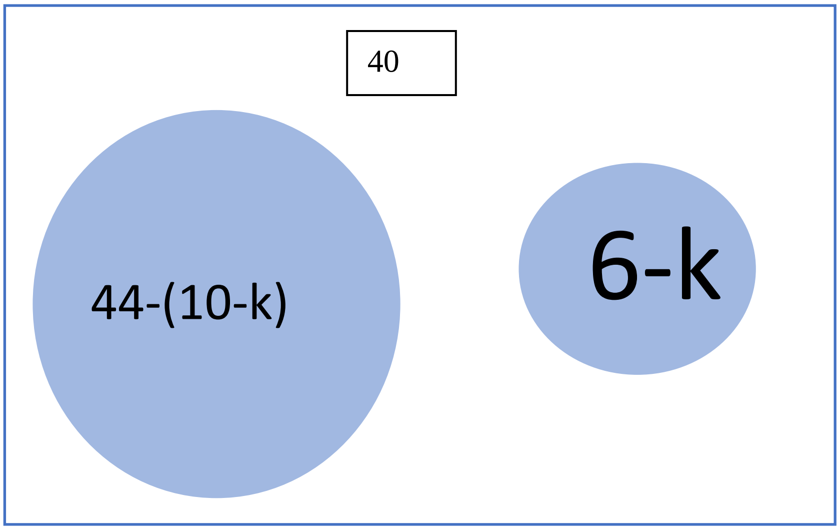


In this diagram the big circle with a 44 represents the tickets that do not contain the prize and the circle with the 6 represents the tickets that do have the prize. The 50 in the box just represents the total amount of tickets, which is the sum of the two circles.

So, in order to calculate the probability that Terri wins k tickets we need to count the number of combinations where Terri wins k tickets. So let's say Terri wins k tickets, that means the we take a k out of the circle that contains a 6, which would result in this diagram



So, from 6 we choose k tickets, this can be represented as $\binom{6}{k}$. Furthermore, since Terri is buying 10 tickets, we will take 10-k tickets from the 44 circle, since if Terri took k tickets that contain a prize, and he bought 10 tickets, then the rest of the 10-k tickets come from the 44 circle, which are the tickets that do not contain a price. The diagram would end up like this



And this can be represented as $\binom{44}{10-k}$ since we are choosing 10-k tickets from the 44 tickets that do not have a price. (Note that we get 40 in the box, since 50-k-(10-k) = 50-k-10+k = 40)

So we multiply these two to get the total amount of combinations of choosing k prices having bough 10 tickets so we get

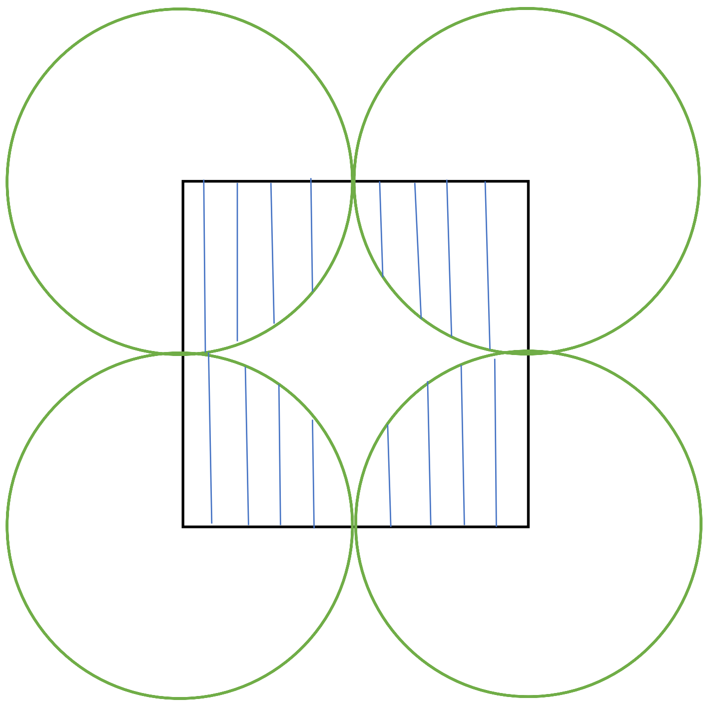
$$\binom{44}{10-k}\binom{6}{k}$$

To finish we divide this by the sample space to get the probability of getting k prices having bought 10 tickets and the sample space is $\binom{50}{10}$, since that is the amount of ways of to pick 10 tickets out of the 50 total tickets. So, the final result is

$$\frac{\binom{44}{10-k}\binom{6}{k}}{\binom{50}{10}}$$

3) **A point is chosen at random inside of a unit square. What is the probability that the point is farther than half a unit from any of the four corners of the square?**

So, a unit square is a square with lengths of 1. Therefore, the area is 1^2 which is 1. Now a point is chosen at random inside the square and we want to find the probability it is farther than 0.5 units from one of the corners of the square. So, we can create 4 circles of radius 0.5 units that are each centered at the corners of the squares in order to represent the area of where the point would be within 0.5 units from one of the four corners. The area of the circles inside of the square would represent this mentioned area since the radius of a circle is always the same and everything inside that circle would less than the length of the radius away from the center of the circle. A diagram would look like this.



The area with the blue lines, where the are of the circles and the are of the square intersect is the area where the point would be within 0.5 units from any of the four corners. So the area inside the square that does not have the blue lines would be the are were the point would be farther than 0.5 units away from any of the 4 corners. So, to find the probability that the point will be farther than 0.5 units from any of the corners, we need to divide the are inside the square without the blue lines by the area of the square, to get what percentage of the total area (square are) this are without the blue lines is. Let's call this area A. since the are of the square is 1, A/1 is just A and therefore we just need to find A. One way we can do this is by subtracting the area of that contains the blue lines from the are of the square. Let's call this area B and Let's call the area of one circle C. Since the radius of the squares is 0.5 units and the center of the circles are the corners of the square, the area of one circle inside the square is the area of the circle divided by 4. The lines of the square inside of the circle are 90 degrees and represent the radius, and therefore it is one quadrant of the circle which is ¼ the are of the whole circle, meaning the area is C/4. Since there are 4 of these, to get B we multiply $\frac{C}{4} \cdot 4$. So

$$B = \frac{C}{4} \cdot 4$$
$$B = C$$

So we have found that B (the total are of the sections with blue lines) is equal to the area of one of the circles. The area of one of the circles is the following.

$$C = \pi r^2$$
$$C = \pi(0.5)^2$$
$$C = \frac{\pi}{4} \qquad (Note: \ 0.5^2 = \frac{1}{2}^2 = \frac{1}{4})$$

$$C = B \qquad and\ therefore\ B = \frac{\pi}{4}$$

So the probability that the random point is farther than half a unit from any of the four corners of the square is $1 - \frac{\pi}{4}$ (area square minus B)

4) **Sam's probability of getting an A on an individual test is 85%. If he takes 12 tests, what is the probability he gets As on exactly 10 of those tests?**

Sam takes 12 tests and he gets A's in 10 of those tests. The probability of him getting an A on any individual test is 0.85. So, the probability he gets A's 10 times would be 0.85 multiplies by itself 10 times since the probability of him getting an A is independent of whether he got an A or not in the previous tests. So the probability of him getting A's in 10 tests is 0.85^{10}. Since he got A's in 10 of them, he would not get A's in 2 of them. The probability of him not getting an A would be 1 minus the probability of him getting and A, since there is a 1 (100%) probability that he either gets an A or does not get an A since those are the only two options in the universe of grades (A's or not A's which would be B, C, D, F, assuming we use UCF letter grade system). So, the probability of Sam of not getting an A in any individual test is 1-0.85 which is 0.15. So the probability of Sam to not get A's in 2 exams is 0.15^2.

Then the probability let's say that he got A's in the first 10 exams and not A's in the last 2 would be

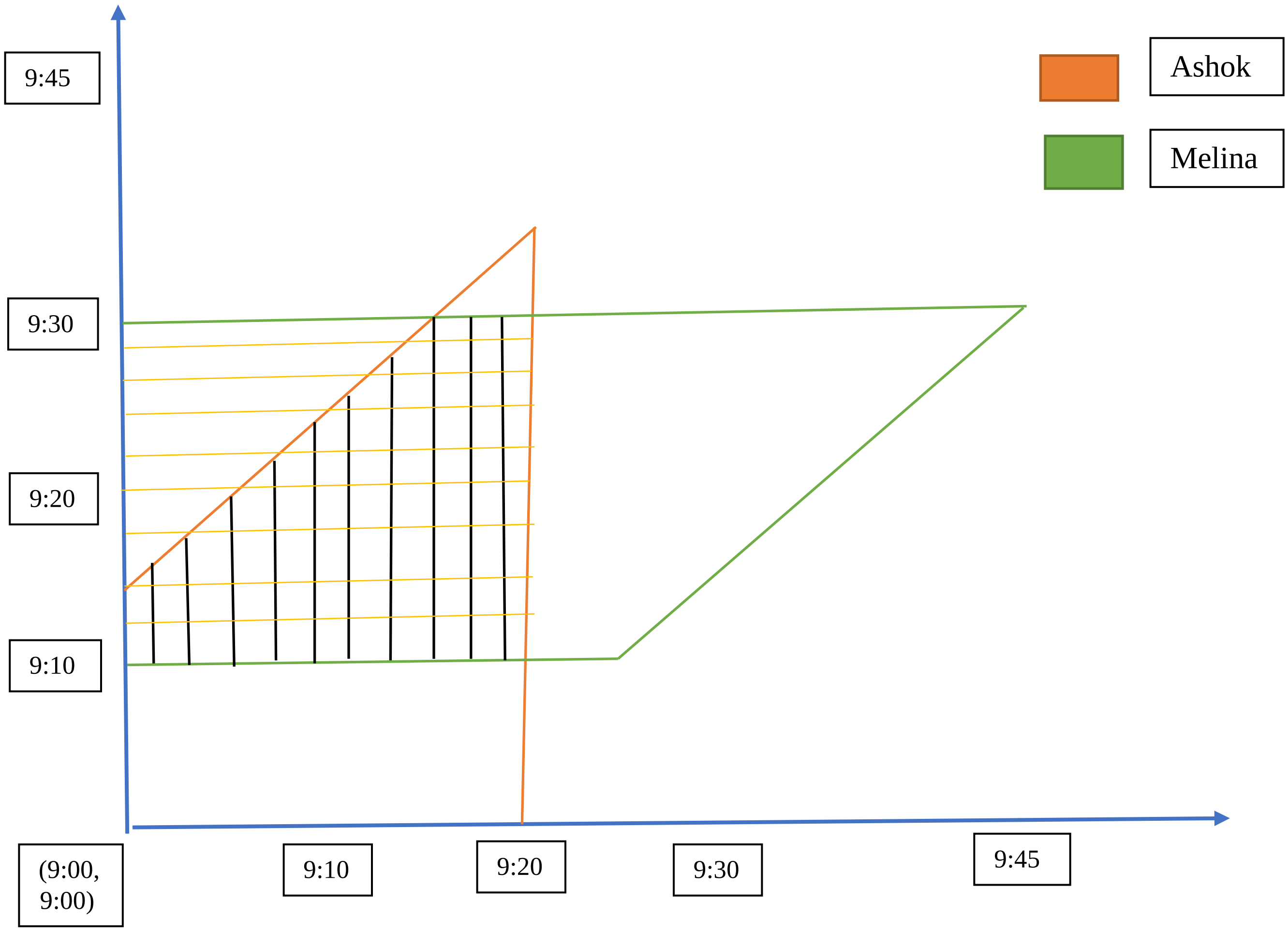
$$0.85^{10} \cdot 0.15^2$$

But we want the probability of him getting 10 A's and 2 not A's in no specific order, so we need to permute the A's and not A's. Since there are 12 total grades and 10 repeated A and 2 repeated non-A's, we would use the formula $\frac{12!}{10!2!}$ which is the same as $\binom{12}{10}$. So, the probability of Sam getting 10 A's out of the 12 tests would be

$$\binom{12}{10} \cdot 0.85^{10} \cdot 0.15^2$$

5) **Ashok arrives at Starbucks at a random time in between 9:00 am and 9:20 am and Melina arrives at Starbucks at a random time in between 9:10 am and 9:30 am. Both stay for exactly 15 minutes. What is the probability that the two of them are in the Starbucks at the exact same time?**

For this question we will create a graph to represent the time of arrival and time of departure of Ashok and Melina. The y axis will be the time of arrival of Melina and the time of departure of Ashok, and the x axis will be the time of arrival of Ashok and the time of departure of Meline.



So, as you can see the area enclosed by the green trapezoid are points in time in which Melina would be at Starbucks, while the are enclosed by the orange trapezoid represent points in time in which Ashok would be at the Starbucks. The area enclosed by both the orange trapezoid and the green trapezoid is the points in time when they are both at starbucks. The are enclosed by both the orange trapezoid and the green trapezoid is the are covered with black lines. The whole square covered by yellow lines represents the points in time where they could meet. So, in order to get the are covered by black lined we need to get the area of the whole square and subtract from it the strangle that only has yellow lines in it but not black lines. That triangle represents points where they could meet but do not, because Melina arrived after Ashok left.

One side of the square is 20 minutes, so the area of the square is $20^2 = 400$. The triangle has side lengths of 15 minutes, because the triangle crosses the y axis at 9:15, which is the time Ashok leaved the Starbucks if he arrives at 9, and from 9:15 to 9:30 there are 15 minutes. The side of the triangle parallel to the x axis is also 15 minutes, because if Ashok arrives at 9:15, he would leave at 9:30 and the side of the triangle ends when the green line crosses the orange graph at 9:30 and that represents Ashok leaving and Melina arriving at 9:30. So the are of the triangle is

$$\frac{15^2}{2} = \frac{225}{2}$$

Then the are covered with black lines, which is the area we want to find, would be

$$400 - \frac{225}{2}$$

Now we just need to divide that are by the area of the whole square to get the probability of Ashok and Melina being at the Starbucks at the same time, since the square is where they can meet and therefore the sample space.

Let M = Ashok and Melina are at the Starbucks at the same time.

$$P(M) = \frac{400 - \frac{225}{2}}{400} \cdot \frac{2}{2}$$

$$P(M) = \frac{800 - 225}{800}$$

$$P(M) = \frac{575}{800} = \frac{115}{160} = \frac{23}{32}$$

6) **Jessica is taking an exam. She will continue attempting to take the exam until she passes it. Because she studies, each time she takes the exam, her chance of passing increases. Let p be her probability of passing the exam the first time she takes it. This means that her probability of failing the exam her first time is 1 - p. On her i^{th} attempt, her probability of failing is $(1-\mathrm{p})^{2^{i-1}}$. Thus, if p = .5, if she fails the first time, her chance of failing the second time is just $.5^2 = .25$. If she fails both the first and second time, her chance of failing the third time is just $.5^4 = .0625$. What is the expected number of times Jessica will take the exam, in terms of p?**

The expected amount of times Jessica will take the exam is the sum of the probability of taking the exam i times multiplied by i from i is 1 to infinity, since there is no limit to the amount of times, she can take the exam. Mathematically this can be written as

$$\sum_{i=1}^{\infty} i \cdot p(i)$$
$$or$$
$$\lim_{N\to\infty} \sum_{i=1}^{N} i \cdot p(i)$$

Where i is the amount of times she takes the test and p(i) is the probability she has of taking n tests. So, we need to find p(i). the probability she takes the exam i times is the multiplication of the probability she fails the first time, then fails the second time … all the probability of failing it the i-1 time, multiplied by the probability of passing the ith time, when i is 2 or greater; when i is 1 then the probability of her taking it 1 time is the probability of passing the first time. This is the case, because she will take the exam until she passes, so once she passes she stops taking the exam, so if she passes the third time for example, she will take the exam three times and the probability of her taking the exam three times is

$$(1-p) \cdot (1-p)^2 \cdot (1-(1-p)^4)$$

Because for her to take three exams she needs to fail it twice and then pass it the third time and her probability of failing the first time is $(1-p)$, the second time is $(1-p)^2$ (from the equation given $(1-\mathrm{p})^{2^{i-1}}$) and the probability of passing the third time is $(1-(1-p)^4)$ which is 1 minus the probability of failing the 3rd time. So the probability of taking the exam i times is

$$\prod_{j=2}^{i} (1-p)^{2^{j-2}} \cdot (1-(1-p)^{2^{i-1}})$$

When i is 2 or greater, when i is 1 the probability is p. So we can put this into the summation and we get

$$p + \lim_{N\to\infty} \sum_{i=2}^{N} i \cdot \prod_{j=2}^{i} (1-p)^{2^{j-2}} \cdot (1-(1-p)^{2^{i-1}})$$

Since $(1-(1-p)^{2^{i-1}})$ is a constant we can take it out of the multiplication, so we get

$$p + \lim_{N\to\infty} \sum_{i=2}^{N} i \cdot (1-(1-p)^{2^{i-1}}) \prod_{j=2}^{i} (1-p)^{2^{j-2}}$$

The multiplication $\prod_{j=2}^{i}(1-p)^{2^{j-2}}$ is the same as saying $(1-p)^{2^{2-2}} \cdot (1-p)^{2^{3-2}} \cdot (1-p)^{2^{4-2}} \cdot \ldots \ldots . \cdot (1-p)^{2^{i-3}} \cdot (1-p)^{2^{i-2}}$ and since all of the terms are the same base we can use (1-p) as the base and add all the exponents, which gives us

$(1-p)^{2^{2-2}+2^{3-2}+2^{4-2}+\cdots.+2^{i-3}+2^{i-2}}$ and this is the same as

$$(1-p)^{\sum_{j=2}^{i} 2^{j-2}}$$

So the sum of the exponents as you might have noticed is a geometric series and the equation for the sum of a geometric series with ratio r $\sum_{i=0}^{n} r^i$ is equal to $\frac{1-r^{n+1}}{1-r}$, so in the sum of the exponents r is 2 and in order to make the sum start from j=0, we can change the range from j=0 to j=i-2 and change the expression 2^{j-2} *to* 2^j, you can notice in this summation $2^{2-2} + 2^{3-2} + 2^{4-2} + \cdots. + 2^{i-3} + 2^{i-2}$ that it is the sum of 2^j from j=0 to j=i-2. So we get

$$(1-p)^{\sum_{j=0}^{i-2} 2^j}$$

And now we can express the sum of the exponents as the equation

$$\sum_{j=0}^{i-2} 2^j = \frac{1-2^{i-2+1}}{1-2} = \frac{1-2^{i-1}}{-1} \cdot \frac{-1}{-1} = \frac{2^{i-1}-1}{1} = 2^{i-1}-1$$

So we then plug this back in and we get

$$\prod_{j=2}^{i} (1-p)^{2^{j-2}} = (1-p)^{2^{i-1}-1}$$

So we plug this back in into our sum and we get

$$p + \lim_{N\to\infty} \sum_{i=2}^{N} i \cdot (1-(1-p)^{2^{i-1}}) \cdot (1-p)^{2^{i-1}-1}$$

Now we can transform the p into (1-(1-p) and include it inside the sum since when i = 1, the sum

$$i \cdot \left(1-(1-p)^{2^{i-1}}\right) \cdot\ (1-p)^{2^{i-1}-1} = 1\left(1-(1-p)^{2^{1-1}}\right) \cdot\ (1-p)^{2^{1-1}-1}$$

$$= (1-(1-p)^{1}) \cdot\ (1-p)^{1-1} = (1-(1-p)^{1}) = p$$

So we get

$$\lim_{N\to\infty} \sum_{i=1}^{N} i \cdot \left(1-(1-p)^{2^{i-1}}\right) \cdot\ (1-p)^{2^{i-1}-1} = \lim_{N\to\infty} \sum_{i=1}^{N} (i - i(1-p)^{2^{i-1}}) \cdot\ (1-p)^{2^{i-1}-1}$$

$$= \lim_{N\to\infty} \sum_{i=1}^{N} (i \cdot\ (1-p)^{2^{i-1}-1} - i(1-p)^{2^{i-1}} \cdot\ (1-p)^{2^{i-1}-1})$$

$$= \lim_{N\to\infty} \sum_{i=1}^{N} (i \cdot\ (1-p)^{2^{i-1}-1} - i(1-p)^{2^{i-1}+2^{i-1}-1})$$

$$= \lim_{N\to\infty} \sum_{i=1}^{N} i\,(1-p)^{2^{i-1}-1} - i(1-p)^{2^{i}-1}$$

Now we can separate the both terms into two sums

$$= \lim_{N\to\infty} \sum_{i=1}^{N} i\,(1-p)^{2^{i-1}-1} - \sum_{i=1}^{N} i(1-p)^{2^{i}-1}$$

Now if we set the second sum to match the terms in the first sum

$$\lim_{N\to\infty} \sum_{i=1}^{N} i\,(1-p)^{2^{i-1}-1} - \sum_{i=2}^{N+1} (i-1)(1-p)^{2^{i-1}-1}$$

We notice that the second sum "almost cancels" every term from the first sum except the very first term. The very first term in the first sum is just 1. From there, if we pair up the second term in the first sum with the first term in the second sum, the third term in the first sum with the second term in the second sum, and so forth, we get:

$$1(1-p)^{2^{1-1}-1} + 2(1-p)^{2^{2-1}-1} + \cdots + (N-1)(1-p)^{2^{N-2}-1} + N(1-p)^{2^{N-1}-1}$$

And the 2nd sum is

$$1(1-p)^{2^{2-1}-1} + 2(1-p)^{2^{3-1}} \ldots + (N-1)(1-p)^{2^{N-1}-1} + N(1-p)^{2^{N}-1}$$

And as we can see by subtracting both sums we get

$$1(1-p)^{2^{1-1}-1} + 1(1-p)^{2^{2-1}-1} + 1(1-p)^{2^{3-1}} + \cdots + 1(1-p)^{2^{N-1}-1} - 1(1-p)^{2^{N}-1}$$

Including that extra 1, this can be expressed as

$$1 + \lim_{N\to\infty} \sum_{i=1}^{N} (1-p)^{2^{i-1}-1} - (1-p)^{2^{N}-1}$$

And since $\lim_{N\to\infty} (1-p)^{2^{N}-1} = 0$ (since $(1-p) < 1$, so it is a number that when multiplied by itself becomes smaller, ex let's say 1-p is 0.5 then $0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \ldots$.. becomes smaller and smaller as you multiply it more and more by 0.5 and it approached 0)
Then we can reduce the sum to

$$1 + \sum_{i=1}^{\propto} (1-p)^{2^{i-1}-1}$$

And this sum represents the expected number of tests Jessica will take. (Sorry! I didn't mean to make this problem so hard. I don't actually know how to evaluate this sum.)

7) **Suppose that one person in 1,000 people has a rare genetic disease. There is an excellent test for the disease; 98% of the people with the disease test positive and only 4% of the people who don't have it test positive. What is the probability that someone who tests positive has the disease? What is the probability that someone who tests negative does not have the disease?**

Let D = has the disease
And T = tests positive
P(D) = 0.001
P(T | D) = 0.98 (probability of T given D, or testing positive given you have the disease)
P(T | $\overline{D}$) = 0.04 (probability of T given $\overline{D}$, or testing positive given you don't have the disease)

Find P(D | T) (probability of D given T, or having the disease given you test positive)
Find P($\overline{D}$ | $\overline{T}$) (probability of $\overline{D}$ given $\overline{T}$, or not having the disease given you test negative)

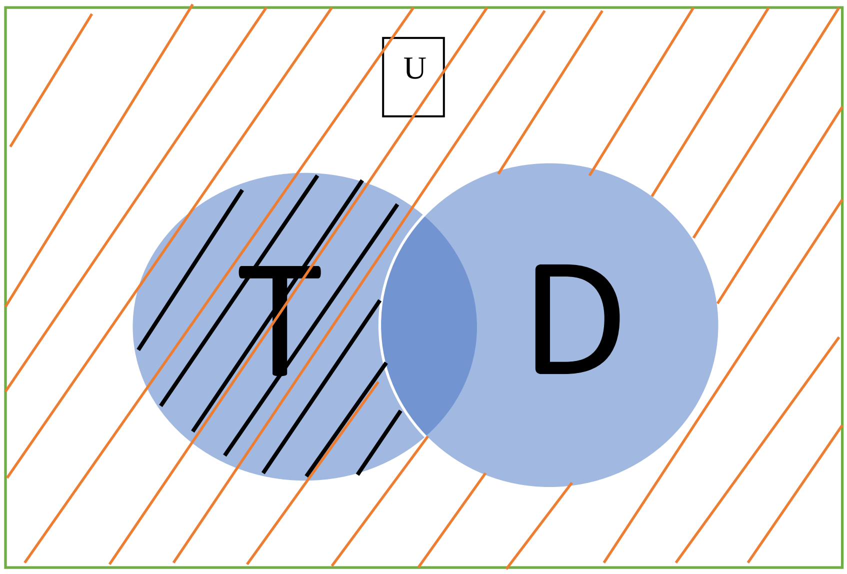
P(D | T) = $\frac{P(D \cap T)}{P(T)}$
P(T | D) = $\frac{P(D \cap T)}{P(D)}$ P(T | D)$\cdot P(D) =$ P(D $\cap$ T) P(D $\cap$ T) $= 0.98 \cdot 0.001$

P(D | T) = $\frac{0.98 \cdot 0.001}{P(T)}$
P(T | $\overline{D}$) = $\frac{P(\overline{D} \cap T)}{P(\overline{D})}$

$P(\overline{D}) = 1 - P(D)$, since, you can either have or not have the disease so the sum of $P(\overline{D}) + P(D) = 1$

P(T | $\overline{D}$) = $\frac{P(\overline{D} \cap T)}{1-0.001}$ P(T | $\overline{D}$) = $\frac{P(\overline{D} \cap T)}{0.999}$ 0.04· 0.999 = P($\overline{D}$ ∩ T)

To relate P($\overline{D}$ ∩ T) and P(T), we can make a Venn Diagram



It is reasonable to assume the Venn diagram looks like this, because $P(T \cap D)$ is not 0 so they intersect, P(T | $\overline{D}$) is not 0 so T is not a subset of D, and $P(T \cap D)$ is not equal to P(D) so D is not a subset of T.
So, the area covered by the orange lines is $\overline{D}$ and the area covered by the black lines is $\overline{D}$ ∩ T and the probability of $\overline{D}$ ∩ T is P($\overline{D}$ ∩ T).
So, |$\overline{D}$ ∩ T| can be written as $|T| - |T \cap D|$ and the probability of that set is $P(T) - P(T \cap D)$.
So, we have
P($\overline{D}$ ∩ T) = $P(T) - P(T \cap D)$
P$(T) - P(T \cap D) = 0.04 \cdot 0.999$
$P(T) = 0.04 \cdot 0.999 + 0.98 \cdot 0.001$

So,

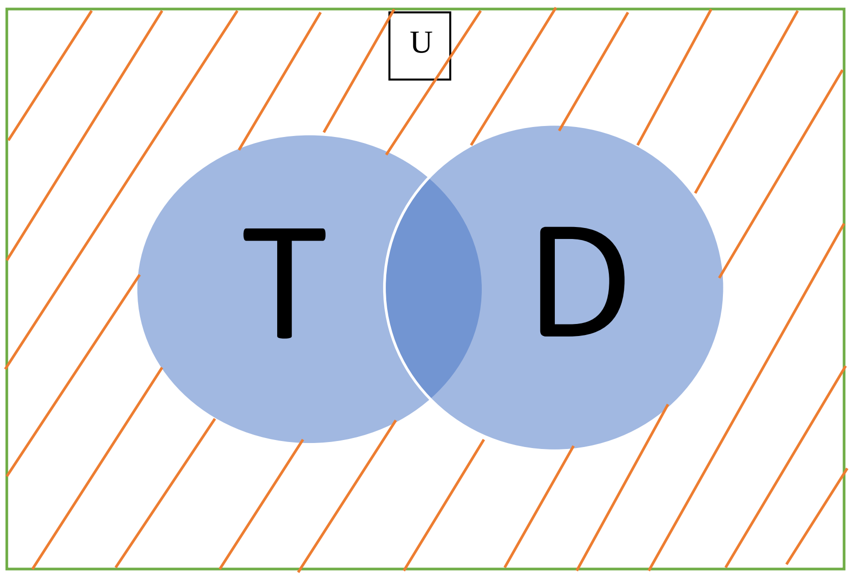
$$P(D \mid T) = \frac{0.98 \cdot 0.001}{0.04 \cdot 0.999 + 0.98 \cdot 0.001} = \frac{\frac{98}{100} \cdot \frac{1}{1000}}{\frac{4}{100} \cdot \frac{999}{1000} + \frac{98}{100} \cdot \frac{1}{1000}} = \frac{\frac{98}{100} \cdot \frac{1}{1000}}{\frac{4 \cdot 999 + 98}{100 \cdot 1000}} = \frac{98}{4 \cdot 999 + 98} = \frac{98}{4 \cdot (1000 - 1) + 98}$$

$$= \frac{99}{4000 - 4 + 98} = \frac{98}{4094} = \frac{49}{2047}$$

Now let's find $P(\bar{D} \mid \bar{T})$,

So, $P(\bar{D} \mid \bar{T}) = \frac{P(\bar{D} \cap \bar{T})}{P(\bar{T})}$

$P(\bar{D} \mid \bar{T}) = \frac{P(\bar{D} \cap \bar{T})}{1 - P(T)} = \frac{P(\bar{D} \cap \bar{T})}{1 - 0.04 \cdot 0.999 - 0.98 \cdot 0.001}$, now we need to find $P(\bar{D} \cap \bar{T})$ and again we can use Venn diagrams for this.



So, the are covered by the orange lines represents $\bar{D} \cap \bar{T}$ and the probability of $\bar{D} \cap \bar{T}$ is just $P(\bar{D} \cap \bar{T})$. $|\bar{D} \cap \bar{T}|$ can be rewritten as $|U| - |D \cup T|$,
$|D \cup T| = |T| + |D| - |T \cap D|$, so $|U| - |D \cup T| = |U| - |T| - |D| + |T \cap D|$ and the probability equivalent of this set is
$P(U) - P(T) - P(D) + P(T \cap D) = 1 - P(T) - P(D) + P(T \cap D) = P(\bar{D} \cap \bar{T})$

So,

$$P(\bar{D} \mid \bar{T}) = \frac{P(\bar{D} \cap \bar{T})}{1 - 0.04 \cdot 0.999 + 0.98 \cdot 0.001} = \frac{1 - P(T) - P(D) + P(T \cap D)}{1 - 0.04 \cdot 0.999 + 0.98 \cdot 0.001}$$

$$= \frac{1 - 0.04 \cdot 0.999 - 0.98 \cdot 0.001 - 0.001 + 0.98 \cdot 0.001}{1 - 0.04 \cdot 0.999 - 0.98 \cdot 0.001}$$

$$= \frac{\frac{100 \cdot 1000}{100 \cdot 1000} - \frac{4}{100} \cdot \frac{999}{1000} - \frac{1}{1000}}{\frac{100 \cdot 1000}{100 \cdot 1000} - \frac{4}{100} \cdot \frac{999}{1000} - \frac{98}{100} \cdot \frac{1}{1000}}$$

$$= \frac{\frac{100 \cdot 1000 - 4 \cdot 999 - 100}{100 \cdot 1000}}{\frac{100 \cdot 1000 - 4 \cdot 999 - 98}{100 \cdot 1000}}$$

$$= \frac{100 \cdot 1000 - 4 \cdot (1000 - 1) - 100}{100 \cdot 1000 - 4 \cdot (1000 - 1) - 98}$$

$$= \frac{100 \cdot 1000 - 4000 + 4 - 100}{100 \cdot 1000 - 4000 + 4 - 98}$$

$$\frac{100 \cdot 1000 - 4096}{100 \cdot 1000 - 4094} = \frac{95904}{95906} = \frac{47952}{47953}$$

8) **Suppose E and F are events in a sample space and p(E) = 1/3, p(F) = 2/5, and p(F | E) = 9/10. Find p(E | F).**

$P(E \mid F) = \frac{P(E \cap F)}{P(F)}$

D $P(F \mid E) = \frac{P(E \cap F)}{P(E)}$, $\quad \frac{9}{10} = \frac{P(E \cap F)}{\frac{1}{3}}$, $\quad P(E \cap F) = \frac{3}{10}$

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{3}{10}}{\frac{2}{5}} = \frac{3}{10} \cdot \frac{5}{2} = \frac{3}{4}$$

9) **Give a summary of the mathematical contributions of Shafi Goldwasser, until now. Please aim for a length of roughly 200 - 400 words. <u>Your summary must be typed.</u> Please state the sources you used in writing your summary.**

Shafi Goldwasser is a professor of computer science at the University of California Berkeley. She also teaches Electrical Engineering and Computer Science at MIT and computer science and is professor of applied mathematics and computer science at the Weizmann Institute of Science in Israel. She has made contributions to cryptography, like coming up with the idea of probabilistic, or randomized encryption, which made encryption safer. She also contributed to the development of a new kind of mathematical proof, which is now called interactive proof system, and which was useful in the creation of cryptocurrency. For her work on computing she won the Turing award. Moreover, she also received the Gödel prize. She is cofounder and chief scientist of Duality Technologies, which is trying to commercialize homomorphic cryptography, which is a mathematical technology that allows one to perform machine learning on encrypted data without first decrypting it.

References

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