

**COT 3100 Fall 2019 Homework #6**  
**Please Consult WebCourses for the due date/time**

1) Let  $H_n$  denote the  $n^{\text{th}}$  Harmonic number. (Recall that  $H_n = \sum_{i=1}^n \frac{1}{i}$ .)

Use mathematical induction on  $n$  to show that  $H_{2^n} \geq 1 + \frac{n}{2}$ , for all non-negative integers  $n$ .

2) Let  $f(n) = \frac{n}{n+2}$ . Define  $f^k(n)$  to be the function  $f$  composed with itself  $k$  times. More formally,  $f^0(n) = n$  and  $f^k(n) = f(f^{k-1}(n))$ , for all positive integers  $k$ . Using induction on  $k$ , prove that for all positive integers  $k$ ,  $f^k(n) = \frac{n}{(2^k-1)n+2^k}$ . (Hint: The algebra can be messy if you don't multiply both your numerator and denominator by  $(2^k - 1)n + 2^k$ . So, in full, after you do a particular step, you would take your fraction and multiply it by  $\frac{(2^k-1)n+2^k}{(2^k-1)n+2^k}$ . Please feel free to ignore the hint, but I do think it reduces the amount of algebra drastically.) (Note: This is an exam question from a previous semester, so a good question for practice!)

3) Consider a rectangular prism with a total surface area of  $94 \text{ in}^2$ . If the sum of all of its edges is 48 in, what is the sum of the lengths of all of its interior diagonals, in inches? Based on the given information, can we determine the exact dimensions of the prism? Why or why not?

4) Let  $P$  be a cubic polynomial such that  $P(0) = k$ ,  $P(1) = 2k$ , and  $P(-1) = 3k$ . What is  $P(2) + P(-2)$ ? Based on the given information, is the polynomial  $P$  uniquely determined? Why or why not?

5) How many permutations of MISSISSIPPI satisfy the following constraints? Consider each case independent of the others.

- a) Have all of its consonants preceding all of its vowels.
- b) Have no two consecutive vowels.
- c) Start and end with a vowel.
- d) Start and end with a consonant.

6) How many ways can we place 8 rooks on a standard 8 by 8 chessboard such that no two rooks can attack one another? (If necessary, please look up the rules of chess to help you solve this problem.)

7) Give a summary of the life and mathematical contributions of Emmy Noether. Please aim for a length of roughly 200 - 400 words. **Your summary must be typed.** Please state the sources you used in writing your summary.