

**COT 3100 Fall 2019 Homework #5**  
**Please Consult WebCourses for the due date/time**

1) Use mathematical induction on  $n$  to prove the following assertion for all positive integers  $n$ :

$$\sum_{i=1}^n \frac{2}{i(i+2)} = \frac{3}{2} - \frac{(2n+3)}{n^2+3n+2}.$$

2) Define the sequence  $t_n$  for all non-negative integers  $n$  as follows:

$$t_0 = 4, t_1 = 13, t_n = 5t_{n-1} - 6t_{n-2}, \text{ for all integers } n \geq 2.$$

Using strong induction on  $n$ , prove for all non-negative integers that  $t_n = 5(3^n) - 2^n$ . (Note: please use 2 base cases.)

3) For all positive integers  $n$ , define the  $n^{\text{th}}$  Harmonic number,  $H_n$ , as follows:  $H_n = \sum_{i=1}^n \frac{1}{i}$ . For all positive integer, using induction on  $n$ , prove that  $\sum_{i=1}^n H_i = (n+1)H_n - n$ .

4) Using proof by induction on  $n$ , prove that  $8 \mid (3^{2n+1} + 5^{2n+1})$  for all non-negative integers  $n$ .

5) Give a summary of the life and mathematical contributions of Evariste Galois. Please aim for a length of roughly 200 - 400 words. **Your summary must be typed.** Please state the sources you used in writing your summary.