

COT 3100 Fall 2019 Homework #4
Please Consult WebCourses for the due date/time

- 1) Find all integer solutions to the equation $1141x + 406y = 28$.
- 2) (a) Find all integer solutions to the equation $105x + 83y = 1$.

(b) Find all integer solutions to the equation $105x + 83y = 8$.

(c) Find $83^{-1} \pmod{105}$. (Note: Answer must be in between 0 and 104, inclusive.)
- 3) Let x and y be integers such that $15 \mid (3x + 4y)$. Prove that $15 \mid (12x + y)$.
- 4) Let $a = 2^4 3^2 5^6 7^8$, $b = 2^2 3^9 5^4 11^5$, and $c = 2^5 3^7 5^3 11^2$. Determine, in prime factorized form, both $\gcd(a, b, c)$ and $\text{lcm}(a, b, c)$.
- 5) For the numbers a , b and c listed in problem 4, determine the number of divisors each of those numbers has.
- 6) How many zeroes are at the end of $\frac{1000!}{500!500!}$?
- 7) The following was proved in class: if p is a prime number and a and b are positive integers such that $p \mid ab$, then either $p \mid a$ or $p \mid b$. Show that this statement is false for composite numbers. Namely, disprove the following statement: if n is a composite number and a and b are positive integers such that $n \mid ab$, then either $n \mid a$ or $n \mid b$.
- 8) In honor of my new license plate (image below), please give a summary of the life and mathematical contributions of Leonard Euler. Please aim for a length of roughly 200 - 400 words. **Your summary must be typed.** Please state the sources you used in writing your summary.

