

COT 3100 Fall 2019 Homework #3
Please Consult WebCourses for the due date/time

1) Let n be a positive odd integer. Prove that $n^4 \equiv 1 \pmod{16}$. You may use any of the mod rules provided in class or the recommended course text book. (In addition, if you end up getting an expression of the form $a(3a+1)$ somewhere in your work where a is an integer, separately end up proving that this expression is always even and use that result to complete your proof.)

2) Convert the following values from the bases indicated to base 10:

i) 3214_7

iv) 20031_4

ii) FCE_{16}

v) 110001010011_2

iii) 35142_8

3) Convert the following base 10 values to the bases indicated:

i) 83111 to base 12

iv) 3426 to base 7

ii) 23650 to base 16

v) 4319 to base 8

iii) 831 to base 2

4) Jessica and Martin start riding their bicycles towards each other at 2 pm. At 2 pm, they are 25 miles apart. Jessica rides her bike at a constant rate of 15 miles per hour and Martin rides his at a constant rate of 10 miles per hour. At 2 pm, a bird starts flying towards Martin. As soon as the bird gets to Martin, it turns back around and flies towards Jessica, and continues going back and forth until Jessica and Martin meet. The bird travels at a constant rate of 45 miles per hour. How far does the bird fly from the time it starts until Jessica and Martin meet? Assume that it takes the bird no time to turn around and fly the other direction.

5) A common divisibility rule is that a positive integer $n = d_k d_{k-1} \dots d_0$, (where each d_i represents a single digit of n), is divisible by 9 if and only if $d_k + d_{k-1} + \dots + d_0$ is divisible by 9. **Discover a somewhat similar divisibility rule for 11 and rigorously prove this via mod rules.**

6) Let x and y be integers such that $17 \mid (3x + 5y)$. Prove that $17 \mid (8x + 19y)$.

7) Give a summary of the life and mathematical contributions of Carl Friedrich Gauss. Please aim for a length of roughly 200 - 400 words. **Your summary must be typed.** Please state the sources you used in writing your summary.