Fall 2019 COT 3100 Section 2 Homework #1

1) Fill out the following truth table:

p	q	r	$\bar{p}$	$r \vee \bar{p}$	$q \wedge (r \vee \bar{p})$	$p \oplus (q \wedge (r \vee \bar{p}))$
F	F	F				
F	F	T				
F	T	F				
F	T	T				
T	F	F				
T	F	T				
T	T	F				
T	T	T				

2) Use the laws of logic to show that the following expression is a tautology:

$$(p \wedge (p \vee (r \wedge s))) \vee (\bar{p} \wedge ((q \vee \bar{s}) \vee s))$$

3) Let r_1 and r_2 be the roots of the quadratic equation $x^2 - 12x + 9 = 0$. Determine the value of $r_1^2 + r_2^2$. (Hint: In the first recitation the relationship between the coefficients in a quadratic and the sum and product of the roots of the quadratic was proven. Use this relationship to solve the problem without finding either root.)

4) Sarah averages 30 miles per hour driving from Orlando to Tampa. What must her average speed be on the return trip if the average speed for the whole round trip was 40 miles an hour? In your work, prove that the answer holds no matter what the distance between Orlando and Tampa is. (Namely, the answer would be true for any two arbitrarily chosen cities.)

5) Show the result of the following bitwise operations. Please show your work (as it's trivial to type these into any programming language and get the result.) In particular, show the conversion of the values to binary, followed by your bit by bit calculation. If you don't know how to do the binary conversion using the proper algorithm by repeatedly dividing by 2, just work it out intuitively adding up powers of 2.

(a) 132 | 85 (b) 217 & 95 (c) 88 ^ 221

6) Explain how to utilize bitwise operators to solve the following problem: All students in a programming club are given a survey of 15 questions, each of the form, "Do you know language X?" Students must answer yes or no to each question. In scheduling lectures, the club would like to avoid lecturing on programming languages that every member in the club knows. For this problem, let's handle the slightly easier question of identifying the number of languages that are known by all members of the club. How can bitwise operators be used to

(a) Store the students' responses to the survey, and
(b) Calculate the answer to the given question?

7) The last question of each homework assignment will be to write up a two paragraph summary of a topic from the history of mathematics. The idea here is that rarely is any of this history taught in mathematics classes and while I don't have class time to teach it, I thought it would be nice if students learned a bit for each homework assignment. There's no need to use fancy sources, websites will do, but please site which websites you pulled your information from.

What are the Millennium Prize problems? Which is the only one of them to be deemed solved? Who is given credit for completing the solution? Give a summary of this person's life and contributions to mathematics.