Outline of Intro to Discrete Exam #2 Topics

I. Number Theory

- **a. Division Algorithm**
- **b. Euclid's Algorithm**
- **c. Extended Euclid's Algorithm**
- **d. Full Solution to ax+by = c for integers given a,b,c.**
- **e. Finding modular inverses**
- **f. Divisibility proofs**
- **g. Pi notation**
- **h. Fundamental Thm of Algebra**
- **i. Least Common Multiple (LCM)**
- **j. Connection between LCM and GCD**
- **k. Idea behind proofs relating lcm, gcd with 3 integers.**
- **l. Calculating # of divisors of an integer.**
- **m. Calculating the sum of divisors of an integer.**
- **n. Calculating the number of times prime p divides into n!**
- **o. Proof that $\sqrt{2}$ is irrational.**

II. Arith Geo Series, Sums, Matrices

- **a. Arithmetic Series - solving for terms, sums, etc.**
- **b. Geometric Series - solving for terms, sums, etc.**
- **c. How to recursively define sequences**
- **d. Definition of summation notation**
- **e. Summation Rules**
- **f. Telescopic sum idea**
- **g. Idea of bounding sums with integrals**
- **h. Matrix Addition, Subtraction, Multiplication**

III. Mathematical Induction

- **a. Base Case**
- **b. Inductive Hypothesis**
- **c. Inductive Step**
- **d. Summation Rules**

e. Not all induction problems use summations
f. How to deal with inequalities
g. Strong Induction
h. Divisibility Problems
i. Matrix Exponentiation Problems
j. Problems with recursively defined sequences
k. Problems with Harmonic numbers
l. Unorthodox Examples - NIM, Nuggets, Trominos

IV. General Math
a. Log problems
b. D = rt problems
c. Arithmetic Sequences

Reading from Textbook

Number Theory: 4.1 - 4.4
Sums, Matrices: 2.4, 2.6
Mathematical Induction: 5.1 - 5.3

What to Study

1) Read the notes.

2) Skim textbook.

3) Practice problems posted online in my archive.

4) Look over written lectures and past homework solutions.

Sample Questions

1) Determine 59^{-1} mod 203. Please express your answer as an integer in between 0 and 202. In order to earn full credit you must use the Extended Euclidean Algorithm.

2) Using induction on n, prove for all non-negative integers n that $9 \mid (2^{2n} + 6n - 1)$.

3) Using induction on n, prove for all positive integers n that $\sum_{i=1}^{n} i^2 \leq n^3$.

4) Let $H_n = \sum_{i=1}^{n} \frac{1}{i}$. Using induction on n, prove for all positive integers n that

$$\sum_{i=1}^{n} iH_i = \frac{n(n+1)}{2} H_n - \frac{n(n-1)}{4}$$

5) Prove for all positive integers n, that in a two player NIM game with two piles of stones, the second player wins if and only if the number of stones in each pile is equal. (On a turn, a player must select a single pile and remove a positive number of stones from it. The winner is the player who removes the last stone.)

6) Prove using induction on n, for all positive integers n, $\sum_{i=1}^{n^2} \sqrt{i} \leq \frac{n(n+1)(4n-1)}{6}$.

7) Let M = $\begin{bmatrix} a & 0 \\ 1 & 1 \end{bmatrix}$, where a is a given positive constant not equal to 1. Prove using induction on n, for all positive integers n, that $M^n = \begin{bmatrix} a^n & 0 \\ \frac{a^n - 1}{a - 1} & 1 \end{bmatrix}$.

8) Prove using strong induction on n with 3 base cases, that if and only if $3 \mid n$, then $2 \mid F_n$. where F_n represents the n^{th} Fibonacci number. (Note: For this question use the following $F_0 = 0$, $F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$, for all integers $n \geq 2$.)

Note: Solutions to these were worked out in class and included in the daily scanned .pdfs of notes on the course grid of notes.