COT 3100 Fall 2018 Homework 10
Please Consult WebCourses for the due date/time.

1) Let R_1 and R_2 be relations on a set A = {1, 2, 3, 4}.
In particular, let R_1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3) (4, 4)} and R_2 = {(1, 2), (2, 3), (3, 4), (1, 3), (2, 4)}.

Determine the following:

a) Whether or not R_1 is reflexive, irreflexive, symmetric, anti-symmetric and transitive or not.
b) Whether or not R_2 is reflexive, irreflexive, symmetric, anti-symmetric and transitive or not.
c) The relation $R_1 \circ R_2$.
d) The relation $R_2 \circ R_1$.
e) $R_1 \cup R_2$
f) $R_1 \cap R_2$
g) The reflexive, symmetric and transitive closures of both R_1 and R_2.

2) Let $A \subseteq Z$ be a subset of integers and let R be a relation over $A \times A$ defined as follows:

$$(x_1, x_2)\ R\ (y_1, y_2) \text{ whenever } x_1 y_2 \leq y_1 x_2$$

Note: This means that ordered pairs in R are of the form ((x_1, x_2), (y_1, y_2)).

Let $A = Z$, determine whether R satisfies each of the following properties. Justify your answers.

(i) Reflexive
(ii) Symmetric
(iii)Transitive

Now, Let $A = Z^+$, determine whether or not R is transitive. Justify your answer.

3) Let b(n) equal the value of the highest bit set to 1 in the binary representation of the positive integer n. (For example, b(27) = 16 because in $27 = 11011_2$ and the most significant bit set to one is the first bit on the left, which has value 2^4.) Prove that the relation, R, defined below over the ***set of integers in between 0 and 1023,*** **inclusive,** is an equivalence relation. Into how many equivalence classes does R partition the set described? Explicitly list all of the members of the following equivalence classes: [2] and [13]. Let the set X be the largest of the equivalence classes. What is the smallest integer that belongs to X?

$$R = \{(x, y) \mid b(x) = b(y)\}$$

4) Let R be a relation over the positive integers defined as follows:

R = {(a,b) | gcd(a,b) > 1 but $a \nmid b$ and $b \nmid a$ }

In laymen's terms, describe how to determine whether or not two positive integers are related via R.

Determine whether or not R satisfies the following properties. Give a brief justification for each of your answers.

(i) reflexive
(ii) irreflexive
(iii) symmetric
(iv) anti-symmetric
(v) transitive

5) How many anti-symmetric relations on the set A = {1, 2, 3, 4, 5, 6, 7} contain the ordered pairs (2, 3), (5, 2), (3, 3), (4, 4) and (6, 6)?

6) Let $f(x) = x^2 - 4x - 48$ with a domain of all real $x \in [-\infty, 2]$. Prove that f is injective. What is the range of f? (You may either use Calculus or complete the square to prove your answers.)

7) Find $f^{-1}(x)$ for the function given in question #6.

8) Let A be a set of 15 elements and B be a set of 12 elements. How many functions can be defined with the domain of A and the co-domain of B?

9) Let $f(x) = \sqrt{17x - x^2}$ and g(x) = 3x + 4, defined over a domain of [0, 17]. Determine f(g(x)) and g(f(x)).

10) Let f(x) = ax + b, where a and b are non-zero constants with $a \neq 1$. Let $f^n(x)$ to be the function f composed with itself n times. (For example, $f^3(x) = f(f(f(x)))$.) Using trial and error, conjecture a guess for $f^n(x)$ and use mathematical induction to prove that guess. Your guess should be a closed form without any summations in it. The constants/variables that should appear in your guess are a, b, n and x.

11) Give a summary of the life and mathematical contributions of Sophie Germain. Please aim for a length of roughly 200 - 400 words. **Your summary must be typed.** Please state the sources you used in writing your summary.