COT 3100 Fall 2018 Homework #7
Please Consult WebCourses for the due date/time

1) Use induction on n to prove the following inequality for all positive integers n:

$$\sum_{i=1}^{2^n} \log_2 i \le (n-1)2^n + 1$$

(Hint : Remember that $\log_2(2^x - y) \le x$ when x is a positive integer and y is a non-negative integer such that $y < 2^x$.)

2) Define a recurrence relation t_n as follows: $t_0 = 1$, $t_1 = 5$, $t_n = 4t_{n-1} - 3t_{n-2}$, for all $n \ge 2$. Using induction on n, prove that $t_n = 2(3^n) - 1$.

3) Let H_n denote the n^{th} Harmonic number. (Recall that $H_n = \sum_{i=1}^{n} \frac{1}{i}$.)

Use mathematical induction on n to show that $H_{2^n} \ge 1 + \frac{n}{2}$, for all non-negative integers n.

4) Let $f(n) = \frac{n}{n+2}$. Define $f^k(n)$ to be the function f composed with itself k times. More formally, $f^0(n) = n$ and $f^k(n) = f(f^{k-1}(n))$, for all positive integers k. Using induction on k, prove that for all positive integers k, $f^k(n) = \frac{n}{(2^k-1)n+2^k}$. (Hint: The algebra can be messy if you don't multiply both your numerator and denominator by $(2^k - 1)n + 2^k$. So, in full, after you do a particular step, you would take your fraction and multiply it by $\frac{(2^k-1)n+2^k}{(2^k-1)n+2^k}$. Please feel free to ignore the hint, but I do think it reduces the amount of algebra drastically.) (Note: This is an exam question from a previous semester, so a good question for practice!)

5) Give a summary of the life and mathematical contributions of Paul Erdos. Please aim for a length of roughly 200 - 400 words. ***<u>Your summary must be typed.</u>*** Please state the sources you used in writing your summary.