

**COT 3100 Fall 2018 Homework #2**  
**Please Consult WebCourses for the due date/time**

- 1) For all integers  $n$ , prove that  $n(3n+1)$  is an even integer.
- 2) Let  $n$  be an odd integer. Using the result from question #1, prove that  $16 \mid (n^4 - 1)$ . (Note: this question does require you to multiply out an expression of the form  $(x+y)^4$ . The algebra is not as bad as it looks...also you can apply the binomial theorem to speed up the work. Google it if you haven't seen it before.)
- 3) Given a set of  $n$  positive real numbers  $a_1, a_2, \dots, a_n$ , with an average of  $b$ , prove that the value of the largest element in the set is strictly less than  $bn$ .
- 4) Let  $S = \{1, 4, 7, 9\}$  and  $T = \{1, 2, 7, 8\}$ . Explicitly list the members of the following sets:  $S \cup T, S \cap T, S - T, S \times T, T \times S, \wp(S)$  and  $\wp(T)$ .
- 5) Use set laws to prove that the two following sets are equivalent.

$$(1) A \cup B$$

$$(2) (A \cap B) \cup (A \cap \bar{B}) \cup (\bar{A} \cap B)$$

- 6) Let  $A, B$  and  $C$  be arbitrary sets taken from the positive integers.

Prove or disprove: If  $A \cap B \cap C = \emptyset$ , then  $(A \subseteq \bar{B}) \vee (A \subseteq \bar{C})$ .

- 7) Give a summary of the life and mathematical contributions of Augustus De Morgan. Please aim for a length of roughly 200 - 400 words. **Your summary must be typed.** Please state the sources you used in writing your summary.