Fall 2018 COT 3100 Section 2 Homework #1

1) Fill out the following truth table:

p	q	r	$(\neg q \vee r)$	$\neg(\neg q \vee r)$	$(p \wedge \neg(\neg q \vee r))$	$p \wedge q$	$(p \wedge \neg(\neg q \vee r)) \vee (p \wedge q)$
F	F	F					
F	F	T					
F	T	F					
F	T	T					
T	F	F					
T	F	T					
T	T	F					
T	T	T					

2) Use the laws of logic to prove the two following expressions are logically equivalent:

(a) $p \wedge (\overline{\bar{r} \wedge \bar{p}} \wedge (\bar{p} \rightarrow s))$ (b) p

3) Use the laws of implication to complete the following argument:

1. $p \vee q$
2. $p \rightarrow s$
3. $q \rightarrow r$
4. $\bar{r}$
5. $s \rightarrow (t \wedge u)$

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t

(Note: Numbers 1 – 5 are the premises and proposition below is what is to be deduced from those premises.)

4) Use the rules of propositional logic and laws of inference to prove the following argument:

1. $\neg((\neg p \vee \neg p) \wedge (\neg p \vee \neg q))$
2. $\neg p$

$\neg r$

(Note: Numbers 1 – 2 are the premises and proposition below is what is to be deduced from those premises.)

5) Prove or disprove the following statements. Use the domain of real numbers for each variable, unless otherwise stated in the problem.

a) $\exists x[x^2 + 4x + 3 > 2x^2 + 5x + 6]$
b) $\forall x \in Z^+[(x + 1)^2 > x^2]$
c) $\exists x[\forall y(y^2 - 3xy + x^2 \geq 0]$
d) $\exists x[\forall y(y^2 - 3xy + x^2 > 0]$
e) $\exists x \in Z^+[\exists y \in Z^+(x^y \; has \; precisely \; 7 \; divisors)]$

6) A wonderful advance in theoretical computer science was the Cook-Levin Theorem, discovered in the early 1970s. Both Cook and Levin independently discovered that the Satisfiability Problem was NP-Complete. Clearly define and summarize the Satisfiability Problem and give one specific instance of a Boolean expression that is satisfiable and another specific instance of a Boolean expression that isn’t satisfiable.