Important Fall 2018 COT 3100 Final Exam Information

Exam Date and Time
Date: Thursday, December 6, 2018
Time: 10:00 am – 12:50 pm **Location: PSY-108**

NOTE THE EARLIER START TIME!!!

WHAT TO BRING

1. Scantron Readable Pen or Number 2 pencil
2. Raspberry Scantron (see picture posted in Webcourses)
3. Four sheets of pre-prepared notes, either handwritten or typed, used only for part B.

Exam Aids:

Part A: No AIDS

Part B: Four sheets of notes (regular 8.5”x11” paper, front and back either handwritten or typed), plus Exam #1 Formula Sheet (for Part B only). The latter will be provided for you. Please do NOT bring your own copy.

Exam Format

Part A: 10 AM - 11 AM (One Hour Time Limit)
If you come late, you still have to turn it in at 11 am.
50 points - 25 multiple choice questions

Part B: 11 AM - 12:45 PM
If you finish Part A early, you can use more time on this part. It gets collected at 12:45 pm regardless of when you started it.
75 points - several proof-like questions

COURSE TOPICS: GENERAL MATH - ALGEBRA, ETC. (BASED ON WARM-UPS), LOGIC, PROOF TECHNIQUES, SETS, NUMBER THEORY, SUMS, MATRICES, INDUCTION, COUNTING, PROBABILITY, RELATIONS, FUNCTIONS

SARC REVIEW SESSION - KATHRYN WYRICK
DATE: SUNDAY, DECEMBER 2nd
TIME: 1:30 - 3:30 pm, LOCATION: KEY WEST CD (SU 218)

SARC REVIEW SESSION - SOLIMAN
DATE: TUESDAY, DECEMBER 4th
TIME: 1:30 - 3:30 pm, LOCATION: KEY WEST AB (SU 218)

https://ucfstudyunion.wordpress.com/computer-science-2/

In Class Contest Questions and Solutions

1) What is the average of 2018 integers whose sum is 2018^3? Express your answer in prime factorized form. (Hint: 1009 is prime.)

The average of a set of numbers is the sum of those numbers divided by the number of numbers. Applying that formula here, we get:

$$Avg = \frac{2018^3}{2018} = 2018^2 = 2^2 1009^2$$

2) How many divisors does the 40,000 have?

40,000 = 4 x 10^4 = 2^2 x 2^4 x 5^4 = 2^6 x 5^4. Using the formula derived in class, this number has (6+1)(4+1) = 35 divisors.

3) What is the value of the following summation?

$$\sum_{i=1}^{n} log_{n!} i$$

$$\sum_{i=1}^{n} log_{n!} i = log_{n!}\left(\prod_{i=1}^{n} i\right) = log_{n!} n! = 1$$

The key here is applying the rule for adding logs and recognizing that the desired product is n!

4) In an arithmetic sequence $a_{72} + a_{112} = 22$, what is the sum of the first 183 terms of the sequence?

With only one piece of information, we see that the value of d, the common difference, is not unique. In these situations, multiple different arithmetic sequences are possible, but each of these possible sequences gives the same result for the question posed. The key here is taking the given information and reformulating it so that d disappears. The term in the sequence "halfway" in between a_{72} and a_{112} is a_{92}. Rewrite both of the terms in the sum in terms of a_{92}:

$a_{72} = a_{92} - 20d$
$a_{112} = a_{92} + 20d$

Adding these two equations, we get $22 = a_{72} + a_{112} = 2a_{92}$, so $a_{92} = 11$.

Now, let's rewrite the sum desired in terms of a_{92} and d:

$a_1 + a_2 + \ldots + a_{182} + a_{183} = (a_{92} - 91d) + (a_{92} - 90d) + \ldots + (a_{92} + 90d) + (a_{92} + 91d)$

Notice that all of the d's cancel out as we can pair up terms equally far away from a_{92} on both sides. It follows that the desired sum is

$$= 183a_{92}$$
$$= 183 \times 11$$
$$= 2013$$

5) What is the greatest common divisor of 234 and 91?

234 = 2 x 91 + 52
91 = 1 x 52 + 39
52 = 1 x 39 + 13
39 = 3 x 13

Thus, the desired GCD is 13.

6) 27 students own a car, bicycle or skateboard. 7 students own both a car and bicycle, 8 students own both a car and skateboard, and 9 students own both a bicycle and skateboard. 3 students own all 3. Out of the 27 students, the same number own a car as own a bicycle as own a skateboard. How many students own a car in the group?

Let X be the number of students who own a car. Note that this is also the number of students who own a bicycle, and who own a skateboard. Let the sets A, B and C, denote these groups, respectively. Using the general Inclusion-Exclusion Principle, we have the following equation:

$$|\boldsymbol{A} \cup \boldsymbol{B} \cup \boldsymbol{C}| = |\boldsymbol{A}| + |\boldsymbol{B}| + |\boldsymbol{C}| - |\boldsymbol{A} \cup \boldsymbol{B}| - |\boldsymbol{A} \cup \boldsymbol{C}| - |\boldsymbol{B} \cup \boldsymbol{C}| + |\boldsymbol{A} \cap \boldsymbol{B} \cap \boldsymbol{C}|$$
$$\mathbf{27 = 3}\boldsymbol{X} \mathbf{- 7 - 8 - 9 + 3}$$
$$\mathbf{48 = 3}\boldsymbol{X}$$
$$\boldsymbol{X} \mathbf{= 16}$$

7) How many different strings of 6 letters contain either the letter 'a' or the letter 'b'? For the purposes of this problem, there are 26 possible letters.

There are 26^6 total strings. Of these 24^6 don't contain 'a' or 'b' (since there are 24 letter choices if we exclude these two.) It follows that there are $26^6 - 24^6$ strings that contain an 'a' or a 'b'.

8) Let A and B be sets such that |A| = 5 and |B| = 7. How many injective functions can be defined with a domain of A and a co-domain of B?

a) 35 **b) $\frac{7!}{2}$** c) 7! **d) 7^5** e) None of the Above

Solution
Answer: B

The first item in the domain can map to any of the seven values in the co-domain. Since the function is injective, the second item in the domain can only map to any of the six remaining values, and so forth. Thus, using the multiplication principle, we get 7 x 6 x 5 x 4 x 3. The answer choice equal to this is B, because we can multiply the answer by the fraction $\frac{2!}{2}$, leaving it unchanged, to reveal the answer as $\frac{7!}{2}$five times to get 7^5. Thus, the correct answer choice is B.

9) Here is a description of a discrete random variable X:

X = 2, with probability $\frac{1}{6}$
5, with probability $\frac{1}{3}$
6, with probability $\frac{1}{2}$

What is the variance of X?

a) 1 **b) 2** c) 3 d) 5 e) None of the Above

Solution
Answer: B

To find the variance of X, let us determine the Expected Value of X:$2 * \frac{1}{6} + 5 * \frac{1}{3} + 6 * \frac{1}{2} = 5$

The variance of X can be found with $\sum_{x \in X}(p(x)(x-5)^2) = \frac{1}{6}(2-5)^2 + \frac{1}{3}(5-5)^2 + \frac{1}{2}(6-5)^2 = \frac{9}{6} + \frac{1}{2} = 2$

Alternatively, we can find $E(X^2) = \frac{1}{6} * 4 + \frac{1}{3} * 25 + \frac{1}{2} * 36 = 27$

And then calculate $Var(X) = E(X^2) - [E(X)]^2 = 27 - 25 = 2$.