Fall 2018 COT 3100 Exam #2 (10/23/2018) (Note: Out of 100 points) - Pages 1, 2

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Lab Section: 18(R9) 19(R10) 20(R11) 21(T2) 22(T3) 23(T4) 24(T5)

1) (8 pts) What is the result of the following matrix computation?

$$\begin{bmatrix} 9 & 6 & 3 \\ 7 & 4 & 7 \end{bmatrix} \times \begin{bmatrix} 10 & 2 \\ 5 & 7 \\ 10 & 3 \end{bmatrix} =$$

2) (10 pts) What is a closed-form expression in terms of the positive integer *n* for the summation below:

$$\sum_{i=n}^{2n} \left(\sum_{j=1}^{2i} ij \right)$$

3) (14 pts) Prove using induction on n that for all positive integers n, $11 \mid (13^n - 2^n)$.

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4) (15 pts) Find all integer solutions to the equation $177x + 78y = 18$.

5) (15 pts) Find the ***sum of the divisors*** of 225,000, leaving your answer in ***prime factorized form.*** (Hint: First prime factorize the given integer. Use the formula from class to express the sum of divisors as a product of some fractions. It may be helpful for you to use the following factoring formula: $x^3 - 1 = (x - 1)(x^2 + x + 1)$. Then, cancel as necessary and express what remains in prime factorized form. This will require a bit of hand calculation, but nothing that you calculate by hand should exceed 1,000. It also may help you to know that $5^4 = 625$.)

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6) (10 pts) Let t_n be defined as follows: $t_0 = -1$, $t_1 = 2$, $t_n = 5t_{n-1} - 6t_{n-2}$, for all integers $n \geq 2$. Prove, using strong induction on n, that for all non-negative integers n, $t_n = 4(3^n) - 5(2^n)$.

7) (10 pts) Let a be a positive real number with a ≥ 2. Using induction on n, prove for all non-negative integers n that

$$\sum_{i=0}^{n} a^i < a^{n+1}$$

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8) (8 pts) Let a_1, a_2, a_3, … form an arithmetic sequence with $a_{10} = 13$ and $a_{30} = 53$. Determine the sum of the first 20 terms of the sequence, $\sum_{i=1}^{20} a_i$. Put a box around your final answer.

9) (8 pts) Prove for all positive integers a, b and c: if a = gcd(b, c), then $a^2 \mid (bc)$.

10) (2 pts) Which office supply company bought the naming rights to the Staples Center in LA?

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